THE ELIMINATION OF SPURIOUS TRENDS IN MARINE WIND DATA
USING PRESSURE OBSERVATIONS

RALF LINDAU*
Meteorological Institute of Bonn University, Bonn, Germany

Received 4 October 2005
Revised 4 October 2005
Accepted 4 October 2005

ABSTRACT

The use of pressure reports from merchant ships provides a reasonable way to correct the trends in marine wind data. However, the appropriate method to derive pressure gradients from the ship data is controversial. Three principal proceedings are analysed: The method of using three simultaneous pressure observations, the method of deriving monthly mean pressure gradients, and the method presented by Lindau (1995). For the first two, it is shown that they are unable to provide reliable reference values for wind speed; if observation triples are used, random errors in the raw data cause systematic errors in the derived pressure gradients. The problem concerning monthly mean pressure gradients is that they are only proportional to the scalar wind, but not to the vector wind. Therefore, these methods are inapplicable for calibration purposes. The method of Lindau (1995) is recommended for a proper elimination of spurious trends in marine wind observations. Copyright © 2006 Royal Meteorological Society.

KEY WORDS: COADS; wind trend; calibration; pressure gradient

1. INTRODUCTION

Wind speed is one of the most important parameters for the determination of marine climate, since wind controls not only the momentum flux between the atmosphere and the ocean but also the exchange of energy through its influence on evaporation and sensible heat flux. The elimination of any inhomogeneity within marine wind data is therefore of fundamental interest for climate studies. The crucial parameter in this context is the mean scalar wind rather than the vector mean wind speed, as the energy fluxes are affected by the former.

Marine wind observations, such as those collected in the comprehensive ocean–atmosphere data set (COADS), (Woodruff et al., 1987), show considerable trends during the last century. That these trends reflect actually a real climate signal cannot be taken for granted, since the observing practices on board merchant ships have changed over time, which may well have introduced spurious trends into the data. In earlier times, when sail ships prevailed, the wind force was defined by the amount of sails a certain type of ship, the so-called Man of War, could carry. For lower winds, the wind force was estimated by the speed of the ship with all sails set (Kinsman, 1969). By about 1930 this definition became more and more problematic because steamers came up, so the old definition was gradually replaced. Since those times, the Beaufort force is increasingly estimated by the state of the sea; one of the first definitions of the new classification was given by Petersen (1927). Even today, a large fraction of wind observations at sea is carried out as Beaufort estimates. However, more and more ships are equipped with anemometers, introducing further inhomogeneities into the time series of the marine wind speed (Cardone et al., 1990).

*Correspondence to: Ralf Lindau, Bonn University, Meteorological Institute, Auf dem Huegel 20, Bonn, Germany; e-mail: rlindau@uni-bonn.de

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As wind is induced by pressure differences, and pressure measurements are carried out routinely on board merchant ships, an independent control parameter suitable for the calibration of wind observations is available. Ward and Hoskins (1996) used mean pressure gradients to calibrate the wind speed. On the basis of the geostrophic wind equation, it is reasonable to argue that mean pressure gradients are proportional to the mean wind speed. But attention! The proportionality is valid only for the mean vector wind, and not necessarily for the mean scalar wind, which is in most cases the targeted parameter. The following equation concretises this problem:

\[
\sqrt{\left(\frac{\partial p}{\partial x}\right)^2 + \left(\frac{\partial p}{\partial y}\right)^2} \propto \sqrt{\langle u \rangle^2 + \langle v \rangle^2} \neq \sqrt{u^2 + v^2}
\]

(1)

In the above equation, temporal averages are denoted by brackets 

\[
\langle \rangle
\]

The first term is the spatial gradient of the temporally averaged pressure field, which is indeed proportional to the mean vector wind given in the second term. The problem occurs during the transition between the second and the third term, which denotes the mean scalar wind speed. The ratio between vector and scalar wind is often referred to as wind steadiness.

It is obvious that mean pressure differences could be used to calibrate the scalar wind only if the steadiness is constant in time. There is no guarantee that this holds true and in Section 4 we will show that it is actually not the case.

That the weak connection between the vector and scalar winds is actually a true climate characteristic and not affected only by random observation errors is also shown at the end of Section 4.

If mean pressure gradients drop out, it is a self-suggesting idea to use triples of simultaneous pressure observations to derive the instantaneous gradient. However, we will show that the magnitude of the gradient might as well be expressed in terms of the variance between the three involved observations. Any observation error acts as additive and would increase this variance. As a consequence, random errors of the pressure observations have a systematic error effect on the derived gradients. Therefore, the temporal constancy of errors would be required for this method, which is not fulfilled. Beyond that, the errors would dominate the signal. Section 3 of this paper is dedicated to the method of observation triples, where its shortcoming is shown in detail.

Lindau (1995) presented a technique to calibrate individual wind pressure differences between pairs of ship observations. Using the relative wind direction referred to the base line between the two ships, it is possible to conclude not only a component but also the total magnitude of the pressure gradient. Section 6 of this paper is reserved for discussing this method.

However, an inherent assumption of all methods using instantaneous pressure differences is that the pressure field between the observations is substantially linear. Otherwise, the measured pressure difference is not representative for the local gradient. In Section 5 we will therefore initially show that the pressure field is actually to a large extent linear, so that instantaneous pressure differences are indeed useful.

2. DATA

Individual ship reports of COADS release 1 were used 10 years ago to establish the wind correction method proposed by Lindau (1995). In the present paper, this method is entirely documented for the first time. A change to more recent data releases of COADS seemed unnecessary, because the principles of the different methods are the focus of this contribution. For assessing the method of mean pressure differences, additional data analyses were necessary. For the sake of equal treatment, the same data, i.e. individual ship observations of COADS release 1, are used. The same is true for the examination of the linearity of pressure fields. However, the seemingly most manifest method to derive pressure gradients, i.e. the method of observation triples, needs no new data analysis for invalidation.

Intentionally, we did not apply any quality control to the data. The effects of random errors will be cancelled out nearly completely (as shown in Section 4.1 for monthly mean wind speed) or their effects will be eliminated by statistical approaches. Trying to diminish the errors by pre-processing the data is not
absolutely necessary. If errors affect the results, a statistical correction is necessary in any case. Correcting the error effects in one step is more efficient. Systematic errors, on the other hand, are very difficult to identify and need a lot of additional information. Such errors form a separate issue and cannot be treated in this paper, which is focussed on the principles of wind trend corrections.

3. THE METHOD OF OBSERVATION TRIPLES

At first glance, the most straightforward way to obtain the geostrophic wind is to use triples of simultaneous pressure observations. However, as the differences between the three pressure measurements are the interesting quantities here, random observation errors will systematically increase the estimated pressure gradient. The use of erroneous measurements will result in spuriously enhanced gradients, whereas pressure observations of high quality will provide lower values, which are in better agreement with the truth. The knowledge of the errors, especially their temporal evolution, is therefore crucial if this method is to be used to calibrate wind observations. In the following pages, the outlined error effect is discussed and quantified. We will show that the pressure gradient can be expressed in terms of the variance between the three observations, which is of course increased by any observational error variance. Finally, we will come to the conclusion that the method using triples of simultaneous pressure measurements is influenced to such an extent by the error effects that this idea has to be discarded.

Consider three simultaneous pressure observations $P_1$, $P_2$, $P_3$, separated by the distance $a$ and arranged in an equilateral triangle as sketched in Figure 1. As only the differences in pressure are crucial, the average of all three measurements

$$P_0 = \frac{P_1 + P_2 + P_3}{3}$$

may be subtracted from each observation so that only the anomalies

$$p_i = P_i - P_0$$

against $P_0$ are considered. In this case, we are able to express one of the observations by the other two.

$$p_3 = -p_1 - p_2$$

Figure 1. Principal sketch of three ship-based pressure measurements, $p_1$, $p_2$, and $p_3$, arranged in an equilateral triangle. The ships are separated by the distance $a$ from each other. $h$ denotes the height on the base line between $p_1$ and $p_2$. 

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The height \( h \) of a triangle can be expressed as:

\[
    h = a \sin(\alpha)
\]

In an equilateral triangle \( \alpha \) is equal to 60° so that

\[
    h = \frac{\sqrt{3}}{2} a
\]

Assuming that the base line of the triangle is orientated in the east–west direction, the zonal component of the pressure gradient is:

\[
    \frac{\partial p}{\partial x} = \frac{p_2 - p_1}{a}
\]

The pressure gradient in meridional direction is measured orthogonal to the base line, i.e. along the height \( h \). At the southern end of \( h \) the pressure is \((p_1 + p_2)/2\) and at the northern end, it is \(-p_1 - p_2\), according to Equation (4). Thus, the meridional pressure gradient is:

\[
    \frac{\partial p}{\partial y} = -\frac{3}{2} \frac{p_1 + p_2}{h}
\]

Substituting \( h \) from Equation (6) yields:

\[
    \frac{\partial p}{\partial y} = -\sqrt{3} \frac{p_1 + p_2}{a}
\]

The square of the horizontal pressure gradient \( g \) is defined as:

\[
    g^2 = \left( \frac{\partial p}{\partial x} \right)^2 + \left( \frac{\partial p}{\partial y} \right)^2
\]

Using Equations (7) and (10), the squared pressure gradient is:

\[
    g^2 = \frac{(p_2 - p_1)^2 + 3(p_1 + p_2)^2}{a^2}
\]

\[
    = \frac{4p_1^2 + 4p_1p_2 + 4p_2^2}{a^2}
\]

Let us now consider the variance \( V \) of a triple of observations. In general, the variance of a sample is:

\[
    V = \frac{1}{n-1} \left( \sum_{i=1}^{n} x_i x_i - \overline{x} \sum_{i=1}^{n} x_i \right)
\]

In the above equation, the common notation with \( n \) being the number of observations and \( \overline{x} \) the mean of the sample is used. In dealing with anomalies, \( \overline{x} \) can be set to zero. Furthermore, we consider here only three observations so that Equation (14) is reduced to:

\[
    V = \frac{1}{2} (p_1^2 + p_2^2 + p_3^2)
\]
Substituting again $p_3$ by Equation (4), one obtains:

$$V = \frac{1}{2} \left( p_1^2 + p_2^2 + (-p_1 - p_2)^2 \right)$$

$$= \frac{1}{2} \left( 2p_1^2 + 2p_1p_2 + 2p_2^2 \right)$$

$$= p_1^2 + p_1p_2 + p_2^2$$  \hspace{1cm} (17)

The comparison of the Equations (13) and (18) yields:

$$V = \frac{a^2}{4} g^2$$  \hspace{1cm} (19)

or expressed in terms of the standard deviation $\sigma_p$:

$$\sigma_p = \frac{a}{2} g$$  \hspace{1cm} (20)

$V$ can be regarded as the signal, which is measured to determine the gradient. The problem here is that any random error variance produces a systematic spurious increase of $V$. Equation (20) shows further that larger distances $a$ between the ships increase the signal. However, if the distances chosen are too large, the pressure fields cannot be regarded as linear, so the method would fail. A distance of about 200 km is therefore an appropriate ship separation. As the pressure gradient is in the order of $10^{-3}$ Pa m$^{-1}$ the signal $\sigma_p$ will be in the order of 1 hPa.

Kent and Berry (2004) determined the random observation error of COADS pressure measurements to about 2 to 3 hPa, decreasing from 1970 till date with a superimposed spatial variation. This observation error produces on average an increase of $V$ of about 4 to 9 hPa$^2$, and this occurs not by a constant offset but depending on time and location. The signal to be measured is only about 1 hPa$^2$, as estimated in the previous paragraph.

We can conclude that geostrophic winds, which are calculated from triples of individual pressure measurements, are not suitable to serve as calibration for wind observations. As it is the difference in pressure that determines the geostrophic wind, the variance of pressure is the signal here. Any observation error therefore increases the signal systematically. Beyond that, the errors are considerably larger than the true expected signal, and differ temporally and spatially. Therefore, the method of using triples of pressure reports as the base of a calibration for wind observations is not feasible.

4. THE METHOD OF MEAN PRESSURE DIFFERENCES

Spatial gradients of sea level pressure are found to be suitable and useful to check wind trends occurring in the time series of marine ship reports. The geostrophic wind equation gives a relationship between the pressure gradient and the wind speed:

$$u_g = -\frac{1}{f\rho} \frac{\partial p}{\partial y}$$  \hspace{1cm} (21)

$$v_g = +\frac{1}{f\rho} \frac{\partial p}{\partial x}$$  \hspace{1cm} (22)

Assuming that this relationship remains constant over the years, spurious parts of the wind trend can be eliminated. Ward and Hoskins (1996) proposed a more sophisticated approach. They considered the balanced friction flow (BBF), in which the effect of friction is included additionally. However, the basic idea remains unchanged: the wind is forced by the pressure gradient so that both have to be proportional.
For the instantaneous case, the scalar wind speed can be directly arrived at from the two horizontal wind components in Equations (21) and (22). However, as shown in the previous section, the calculation of instantaneous pressure gradients is problematical. Therefore, the geostrophic wind equations (or that for BBF) are often averaged in time, typically over 1 month, and in space, typically over $10^\circ \times 10^\circ$ in latitude and longitude. In this way, mean pressure differences in meridional and zonal direction are obtained. They are a measure for the mean east and north wind components, averaged over the considered time and space frame, but not necessarily for the mean strength of the scalar wind. (It is obvious that the mean vector wind speed is always equal to or smaller than the mean scalar wind speed, since any change in wind direction, whether temporal or spatial, reduces the vector wind.)

However, for many applications in climate studies it is just the scalar wind strength that is relevant. It affects the turbulent energy fluxes as well as the wind stress. Thus, if the actual aim is to correct the time series of the mean scalar wind, one has to rely on a constant ratio between the mean scalar wind and the mean vector wind throughout the years. Ward (1992) tried to resolve all doubts that the assumption of the strong coupling between scalar and vector winds could be problematical. He modelled the effect of a perturbed vector wind on the trend in scalar wind and showed that the effect remains limited if the mean vector wind is considerably larger than its standard deviation. However, the assumed perturbation of $0.75 \, \text{m} \, \text{s}^{-1}$ is much too small. The observed standard deviation of the zonal wind, e.g. in $40^\circ \, \text{N}$, amounts to $4 \, \text{m} \, \text{s}^{-1}$ in July and $7 \, \text{m} \, \text{s}^{-1}$ in January (Lindau, 2000). The corresponding magnitudes of the zonal wind speed itself are even smaller, attaining only about one half of these values. So, the standard deviation is not small, but twice as high as the wind speed itself. This holds true for nearly the entire Atlantic, apart from the inner trade wind zones.

We have so far argued that the correction of scalar wind trends by mean pressure gradients is an indirect method. From a change in the mean pressure gradient, one may indeed conclude a corresponding change in the vector mean wind speed, but not necessarily a corresponding change in the scalar wind speed. To discuss the performance of the customary mean pressure method, it is reasonable to consider the directional steadiness of the wind. The directional steadiness $S$ is defined as:

$$
S = \frac{\sqrt{u^2 + v^2}}{\sqrt{u^2 + v^2}}
$$

where $u$ and $v$ denote the zonal and meridional wind components, respectively, and horizontal bars express temporal or spatial averages. It is the inherent assumption of mean pressure methods that the wind steadiness remains constant throughout the years.

Individual wind observations of COADS from the North Atlantic are used to analyse the characteristics of wind steadiness. The main focus is on the temporal variability of the steadiness which is assumed to be negligible in any mean gradient method. Steadiness is defined only for averages of the wind in time or space. In general, the steadiness will decrease if the averaging frames are enlarged. In order to assess the effect of different resolutions, we used two different spatial averaging frames, the first is $1^\circ \times 1^\circ$ and the second $10^\circ \times 10^\circ$. Temporally, the averaging frame is held constant at 1 month. Figure 2 shows the change in wind steadiness for the $10^\circ \times 10^\circ$ field between $40^\circ \, \text{N}$ and $50^\circ \, \text{N}$ and between $30^\circ \, \text{W}$ and $40^\circ \, \text{W}$ in the month of January. The dashed line denotes the year-to-year fluctuation based on the $10^\circ \times 10^\circ$ averages. Here, one value for the steadiness is obtained for each year. The solid line denotes the mean steadiness within the $10^\circ \times 10^\circ$ field based on the 100 individual $1^\circ \times 1^\circ$ values. Through all the years, this estimate is higher than the first, since any change in wind direction within the $10^\circ \times 10^\circ$ field reduces the total steadiness additionally. However, both the time series show similar patterns. The mean steadiness varies considerably between 0.8 and 0.3, and the total steadiness attains in some years even values near zero. The behaviour found in this specific $10^\circ$-field is characteristic for a broader region. The extension of the analysis to the entire $10^\circ$-zone between $40^\circ \, \text{N}$ and $50^\circ \, \text{N}$ in the Atlantic provides similar results (Figure 3); only the total steadiness is further decreasing owing to the larger area considered here, which increases the spatial variety of included wind directions. These results give the first impression that wind steadiness is strongly fluctuating from year to year at least in the mid-latitudes. Assuming its temporal constancy may be not appropriate.
For this reason, a year-to-year correction of the scalar wind by measures for the vector wind – and the mean monthly geostrophic wind is not more than that – is doomed to fail. However, it is still possible that decadal trends in both the scalar and the vector winds are highly correlated, so that at least the trend in the vector wind could be used to correct that in the scalar wind. To check this possibility, we computed wind trends in the North Atlantic for the period 1950 to 1980. The result for the month of May within the area between 40° and 50° North and 40° and 50° West is shown in Figure 4. The trend in the vector wind differs significantly from that in the scalar wind. In fact, both trends are contrary, negative in the scalar wind by 2.3 m s\(^{-1}\) per century, but positive in the vector wind by 3.2 m s\(^{-1}\) per century. In order to show that the coupling between both the trends is generally loose, the method is applied to every month of the year and to seven 10° × 10° fields in the Atlantic between 40°N and 50°N. In this way, 84 trend pairs are available so that a statistical comparison of both is possible. In Figure 6 only the trends themselves, but not the underlying time series, are shown. The comparison of scalar and vector wind trends shows high scatter, meaning that the correlation between both is low. The correlation coefficient is found to be as low as 0.598, implying that less than 36% of variability in the scalar wind can be explained by trends in the vector wind. The use of the vector wind trend as calibration for the scalar wind trend is therefore not possible. Even the total mean trends
are in disagreement. The average trend for all 84 cases does not differ significantly from zero for the scalar wind (0.09 ± 0.26 m s\(^{-1}\) per century), but is significantly positive in the vector wind (0.55 ± 0.36 m s\(^{-1}\) per century).

Mean pressure differences are indeed highly connected with the corresponding vector averages of the wind. However, the connection between vector and scalar winds is very low. The wind steadiness, defined as the ratio between both, is by no means constant in time. Even decadal trends in the scalar and vector winds show, at least in the mid-latitudes, only a low correlation of about 0.6, which is much too small for the use of the vector wind as calibration for the scalar wind.

4.1. Errors of monthly mean winds and their effect on wind trends

We discussed so far the steadiness of wind (Figures 2 and 3) and wind trends (Figures 4 and 5) on the basis of monthly 10° × 10° averages. The question that arise is, how reliable these results are, if random observation errors are taken into account.

For this purpose, the used ship data is analysed again, now with the focus on observation errors and their impacts on variability and trends.

First, we will derive the accuracy of monthly mean wind speed (vector and scalar), which will allow us to estimate the error in steadiness. In a second step, the effect of the errors on the trend (again for vector and scalar winds) will be assessed.

4.2. Errors of monthly mean winds

4.2.1. Scalar wind. For the particular 10° × 10°-field and month shown in Figure 4 (May 40°N–50°N, 40°W–50°W), we calculated some statistics of the scalar wind to answer the above question. The total number of wind observations within the entire field between 1950 and 1979 is 41,667. The total variance is equal to 16.51 m\(^2\) s\(^{-2}\).

According to Lindau (2003), the total variance can be decomposed into two variance parts and two error variance parts: the variance of monthly means (of a 10° × 10° area) and the mean variance within a time span of 1 month (and a 10° × 10° area) plus the error variance of the total mean and the mean error variance of monthly means. We intend to know the last expression here. It amounts to 0.01 m\(^2\) s\(^{-2}\), so that the error of monthly means is 0.1 m s\(^{-1}\).

We can confirm that this error derivation yields the correct value by considering the three other terms of the variance decomposition. The error of the total mean is negligibly small (since 41,667 observations are used). The variance of monthly means is equal to 0.48 m\(^2\) s\(^{-2}\) and the mean variance within a month is 16.03 m\(^2\) s\(^{-2}\). Thus, the intra-monthly variance is typically 16 m\(^2\) s\(^{-2}\). The number of monthly

![Figure 4](https://example.com/figure4.png)

Figure 4. Wind trends for a 10° × 10° field between the longitudes 40°W and 50°W and the latitude 40°N and 50°N for the month of May. The solid line shows the trend for scalar wind, and the dashed line that for the vector wind.
Figure 5. Correlation between the trend in the scalar wind speed and that in the vector wind. Both trends are computed for each month of the year and seven different 10° × 10° fields within the latitudinal strip between 40°N and 50°N in the North Atlantic. The first digit gives the longitudinal number from 1 (70°W–60°W) to 7 (10°W–0°), the second character denotes the month from a for January to l for December.
Figure 6. Time series of the averaged cosine of the errors in wind direction. The mean wind speed is underestimated by this factor \( f = \langle \cos \Delta d \rangle \). Monthly averages of \( f \) are shown for the zone between 40°N and 50°N in the Atlantic for the period 1890 to 1980. \( f \) is on average about 0.9, with a negative trend of \(-0.0619\) cty\(^{-1}\). The uncertainty of the linear trend is \( \pm 0.0039\) cty\(^{-1}\).

\[
\langle U \rangle = - (w + \Delta w)(\sin d \cos \Delta d + \cos d \sin \Delta d)
\]

with \( \Delta w \) and \( \Delta d \) being the errors in wind strength and direction, respectively. Averaging \( \langle \rangle \) with the assumption of random errors leads to:

\[
\langle U \rangle = \langle - w \sin d \rangle \langle \cos \Delta d \rangle
\]

\[
\langle U \rangle = \langle u \rangle \langle \cos \Delta d \rangle
\]

Thus, the mean zonal wind component is underestimated by the factor \( \langle \cos \Delta d \rangle \), denoting the mean of the cosine of the error in the wind direction, which is not zero. An analogous consideration shows that the same is true for the meridional wind component, so that in total the vector wind is underestimated by the factor \( f = \langle \cos \Delta d \rangle \).

The factor \( f \) can be determined by considering the mean difference in wind direction as a function of distance between two individual ships. By deriving a theoretical value for zero distance, \( f \) can be arrived at as described in detail in Section 6. For the zone between 40°N and 50°N, \( f \) is on average 0.893 for the period 1890 to 1980 (Figure 6). However, \( f \) varies from month to month, showing a standard deviation of 0.037. Furthermore, there is an obvious negative trend in \( f \), showing that historical observations were carried out with higher accuracy.

In the following, the effect of the underestimation factor \( f \) is discussed in two aspects. First, its impact on the accuracy of individual monthly mean vector winds is analysed. Later on, its impact on the decadal trend in vector wind will be discussed. Concerning the first issue, it is obvious that vector winds are on average underestimated by \( f = 0.893 \). The temporal scatter of \( f \) (Figure 6), induces a further uncertainty into the monthly mean vector winds. However, the entire variability of \( f \) cannot be regarded as true, because the derivation method of \( f \) itself is not free of error, so only a part of the inter-monthly variance of \( f \) can be regarded as true. However, to estimate an upper limit of the error effect, we consider here the entire variation of \( f \) as true.
Errors in wind direction cause the calculated vector wind $Vec$ to be equal to the true vector wind $vec$ multiplied by the factor $f$:

$$Vec = f \cdot vec$$

(29)

The error of $Vec$ is:

$$\Delta Vec = \sqrt{f^2 \Delta vec^2 + vec^2 \Delta f^2}$$

(30)

where $f$ is equal to 0.893 (Figure 6). $\Delta vec$ denotes the random errors in $vec$ caused by errors in wind strength. This error is already determined to be 0.1 m s$^{-1}$. The magnitude of the mean vector wind is typically 3 m s$^{-1}$ (see Figure 4) and $\Delta f$, the error of $f$, is equal to 0.037, as derivable from the scatter in Figure 6:

$$\Delta Vec = \sqrt{(0.893)^2(0.1 \text{ m/s})^2 + (3.0 \text{ m/s})^2(0.037)^2}$$

(31)

$$\Delta Vec = \sqrt{0.0080 + 0.0123} \text{ m s}^{-1}$$

(32)

Thus, $\Delta Vec$ is increased from 0.09 to 0.14 m s$^{-1}$, if the effects of variability of $f$ are included. This is no substantial increase, especially if we take into account the fact that it is an upper limit.

We can summarise that monthly mean vector winds can be determined with an accuracy of about 0.15 m s$^{-1}$, which is comparable with the error in scalar wind. Their error variances (0.01 and 0.02 m$^2$ s$^{-2}$) are negligible compared to the true inter-monthly variability of 0.48 m$^2$ s$^{-2}$. We have shown that the errors of both the scalar and vector winds are negligibly small. Consequently, the errors in wind steadiness are also small.

4.3. The effects of errors on wind trends

4.3.1. Scalar wind trend. Figure 2 shows that the steadiness in mid-latitudes is about 0.5, varying considerably from year to year. If we accept that such variations of the steadiness reflect true climate variability, it is clear that the scalar wind cannot be corrected by mean pressure gradients, which represent the vector wind. At this stage, the crucial point of this method is already documented. However, one could argue that the steadiness may vary, but that this would have no impact on the trends. The possible upcoming of such an argument leads us to analyse also the trends of the vector and scalar winds. We have showed already that the trends are only weakly correlated, so that a calibration of one by the other is questionable. The question now is: is the observed scatter between the scalar and vector wind trend (Figure 5) caused by natural variability or by observational errors?

By knowing the mean error variance of the underlying monthly means (0.01 m$^2$ s$^{-2}$), we are able to derive the error of the calculated trend. The standard deviation of a regression slope is:

$$\sigma_s = \frac{\sigma_y \sqrt{1 - r^2}}{\sigma_x \sqrt{n - 2}}$$

(33)

where

$\sigma_y$ is the standard deviation of $y$ (wind speed)
$\sigma_x$ is the standard deviation of $x$ (time)
$r$ is the correlation coefficient between time and wind speed
$n$ is the number of observations (30, one for each year)

We can transform the Equation (33) into:

$$(n - 2)\sigma_s^2 = \frac{\sigma_y^2}{\sigma_s^2} - \frac{\text{cov}^2}{\sigma_s^4}$$

(34)
where $\sigma_s$ denotes the variance of the slope instead of the standard deviation, and $cov$ is the covariance between $x$ and $y$. Consider now that we add (to the true natural wind variance $\sigma_y^2$) an error variance $\sigma_e^2$, which is caused by the errors of the monthly means. An additional error variance has no effect on the covariance, so that we can write for the variance of the error-influenced slope:

$$ (n - 2)\sigma_{si}^2 = \frac{\sigma_y^2 + \sigma_e^2}{\sigma_x^2} - \frac{cov^2}{\sigma_x^4} \tag{35} $$

The errors in wind speed cause an additional error variance $\Delta\sigma_{si}^2$, which is equal to the difference $\sigma_{si}^2 - \sigma_s^2$:

$$ \Delta\sigma_{si}^2 = \sigma_{si}^2 - \sigma_s^2 = \frac{\sigma_e^2}{(n - 2)\sigma_x^2} \tag{36} $$

Since the 30 years are equally distributed (and the recording of time can be assumed to be free of error), $\sigma_s$ is equal to 30 years divided by $\sqrt{12}$, or $\sigma_s^2 = \text{cty}^2$. $n$ is equal to 30, and $\sigma_e^2$ is already derived to be 0.01 m$^2$ s$^{-2}$. Thus:

$$ \Delta\sigma_{si}^2 = \frac{0.01 \text{ m}^2 \text{ s}^{-2}}{28 \cdot 0.0075 \text{ cty}^2} \tag{37} $$

$$ = \frac{0.0476 \text{ m}^2 \text{ s}^{-2}}{\text{cty}^2} \tag{38} $$

$$ \Delta\sigma_{si} = 0.218 \frac{\text{m s}^{-1}}{\text{cty}} \tag{39} $$

It is shown so far that observation errors induce an uncertainty of $0.218 \frac{\text{m s}^{-1}}{\text{cty}}$ to the 30-year trends of scalar wind.

For the scalar wind trend itself, presented in the previous section ($-2.3 \text{ m s}^{-1} \text{ cty}^{-1}$, Figure 4), we calculated the standard deviation to be 1.466 m s$^{-1}$ cty$^{-1}$, corresponding to a variance of 2.15 m$^2$ s$^{-2}$ cty$^{-2}$. This means that only 2% of the error variance of the slope is caused by observation errors $2.15 \text{ m}^2 \text{ s}^{-2} \text{ cty}^{-2}$ compared to 0.0476 m$^2$ s$^{-2}$ cty$^{-2}$). The major part (98%) of the trend uncertainty is caused by the strong inter-annual climate variability. With a standard deviation of 1.466 m s$^{-1}$ cty$^{-1}$, the trend is only significant at a level of 90%. However, it is important that the major part of the uncertainty is not caused by observation errors, but by true inter-annual variance. Consequently, the noise is mainly climate noise and is unavoidable.

4.3.2. Vector wind trend. The argument for the vector wind trend is again more complicated, because trends in the errors of wind direction, i.e. trends in the above-introduced factor $f$, may affect the calculations. If we fit a linear trend to the time series of monthly mean vector wind $v$, its evolution in time $v(t)$ can be described by:

$$ v(t) = c + d(t - t_0) \tag{40} $$

where $t$ denotes the time and $t_0$ the middle of the considered time period. $c$ is then the fitted wind speed at $t = t_0$ and $d$ is the linear trend. A typical value for $c$ is $c = 3 \text{ m s}^{-1}$, and the linear trend $d$ is $3.2 \text{ m s}^{-1} \text{ cty}^{-1}$ (see Figure 4). More generally, Figure 5 shows that $d = \pm 3 \text{ m s}^{-1} \text{ cty}^{-1}$ can be assumed as typical value for the vector wind trend (as the mean trend in vector wind of all considered monthly $10^\circ$-fields is as low as $0.55 \text{ m s}^{-1} \text{ cty}^{-1}$, with a standard deviation of 3.27 m s$^{-1}$ cty$^{-1}$).

Consider now that the calculated vector wind $V$ is underestimated by the above-discussed factor $f$, which has a trend too:

$$ V(t) = f(t)v(t) \tag{41} $$

$$ V(t) = (a + bt)(c + dt) \tag{42} $$
where we set for simplicity $t_0 = 0$. $a$ and $b$ denote the fitted mean and trend for the factor $f$. Typically their values are (Figure 6): $a = 0.9$ and $b = -0.06 \text{ cty}^{-1}$. So, the four coefficients in Equation (42) can be estimated by:

$$
a = 0.9 \\
b = -0.06 \text{ cty}^{-1} \\
c = 3 \text{ m s}^{-1} \\
d = \pm 3 \text{ m s}^{-1} \text{ cty}^{-1}
$$

From Equation (42), it follows that:

$$
V(t) = ac + (bc + ad)t + adt^2
$$

Without considering $f$, the linear trend in the vector wind is equal to $d$. When the factor $f$, which also is supposed to have a trend, is taken into account, the vector wind trend is modified to: $bc + ad$.

Thus we may compare the trend, without considering the errors:

$$
d = \pm 3 \text{ m s}^{-1} \text{ cty}^{-1}
$$

with the trend, considering the errors:

$$
bc + ad = -0.18 \pm 2.7 \text{ m s}^{-1} \text{ cty}^{-1}
$$

The typical trend in the vector wind of $\pm 3 \text{ m s}^{-1} \text{ cty}^{-1}$ is reduced in magnitude by 10% (owing to the mean factor $f$ being 0.9) and systematically reduced by $0.18 \text{ m s}^{-1} \text{ cty}^{-1}$ (i.e. decreasing positive trends and enhancing negative trends by this amount).

Consider Figure 6. It illustrates that trends in the scalar wind and those in the vector wind are only weakly correlated. After the above discussion, we are now able to quantify the contribution of the observation errors to the low correlation.

For the scalar wind the argument is straightforward. Errors in monthly mean wind speeds are in the order of $0.1 \text{ m s}^{-1}$, introducing an error variance of about $0.01 \text{ m}^2 \text{ s}^{-2}$ to the data. The inter-annual variability is much higher (about $0.48 \text{ m}^2 \text{ s}^{-2}$). Therefore, the inter-annual variability (e.g. in Figure 4) can be considered as the true climate signal. The uncertainty of the fitted linear trend is nevertheless high (error variance of $2.15 \text{ m}^2 \text{ s}^{-2} \text{ cty}^{-2}$). But observation errors contribute only a small part (0.0476 $\text{ m}^2 \text{ s}^{-2} \text{ cty}^{-2}$). Thus, not observation errors, but the high climate variability is responsible for uncertainties in linear wind trend.

The vector wind trends are subject to similar random errors. Additionally, they are systematically underestimated by a factor of about 0.9 because of errors in the wind direction. The decrease of $f$ through the years yields a third term, imposing a spurious trend reduction of $-0.18 \text{ m s}^{-1} \text{ cty}^{-1}$. However, such a constant dilation and shift of one parameter (the vector wind trend) does not change the correlation (to the scalar wind trend). Furthermore, the magnitude of errors in trend is in the order of tenths of $\text{ m s}^{-1} \text{ cty}^{-1}$ and cannot explain the variation of the trends from month to month and from region to region, which are shown in Figure 5.

We can summarise that the strong variation of trends in the scalar and the vector wind, and the low correlation between both, cannot be explained by errors in the data. The scatter in Figure 5 is in fact largely caused by true climate variability. The low coupling between trends in the scalar wind and that in the vector wind is actually problematic for the method of mean pressure gradients.

5. ARE PRESSURE FIELDS EFFECTIVELY LINEAR?

In the following, the method proposed by Lindau (1995) will be discussed, where the pressure gradient is estimated by pairs of pressure differences, reported simultaneously from two different ships. It is based on the
assumption that the pressure field between the ship measurements is linear. Otherwise the large-scale pressure difference would not be able to represent the local wind. Therefore, we will first verify the assumption of pressure fields being basically linear, before Lindau’s method is presented in detail.

Two principal cases are possible for the basic character of pressure fields, and we want to ascertain the most adequate one. In the first case, the pressure field is indeed spatially linear at each point of time; then its variance will increase quadratically in space. If the second case is true and the field is dominated by stochastic variability, the spatial variance will increase linearly with increasing ship distances. Thus, the first case must be true; otherwise the method is not applicable.

For illustrating the more complex second case, we can consider Brownian motion, which is completely stochastic. If several particles in Brownian motion start at one point, and their distance from the origin is measured as a function of time, the mean squared distance of all particles will increase linearly with time. The constant of proportionality between the mean squared distance and the time has the dimension of m² s⁻¹ and can be regarded as the constant of diffusion, since the spread of particles by Brownian motion can be described as diffusion. This relation between Brownian motion and diffusion is based on considerations of Einstein (1908).

For our purpose, we only have to replace time by space and squared spatial distances by squared pressure differences. Analogously, the squared pressure differences will now increase linearly in space if the pressure field varies stochastically. If, in contrast, the first case is true and pressure differences are caused by a constant pressure gradient – which may of course vary in time, but which is linear for that moment – the squared pressure differences will grow quadratically with increasing ship separation. Returning to the above-used illustration, the Brownian motion used to illustrate the second case is now replaced by a radial divergence of all particles with a constant speed. From this example it is clear that the variance here is growing quadratically.

Most meteorological parameters exhibit strong stochastic dominance. Their variance grows linearly if spatial or temporal distances are increased. The variance of wind direction given in Figure 9 is just one example, where the appropriate linear fit is used later in this paper to determine the observation error. However, the pressure field is different, showing just that feature needed for a reasonable application of the intended method, which is described in the following section. Figure 7 shows the mean squared differences between two marine pressure observations as a function of the distance between the two measuring ships. The increase of variance is clearly a non-linear function, as expected, when the pressure field between the two ships is linear. The fit of a quadratic function

$$\overline{(\Delta p)^2} = a_0 + a_1 \Delta x + a_2 (\Delta x)^2$$

where $\Delta x$ denotes the ship separation and $\Delta p$ the measured pressure difference, yields the following values of the coefficients $a_i$:

$$a_0 = 12.4 \text{ hPa}^2$$
$$a_1 = 13.2 \text{ hPa}^2 \text{ Mm}^{-1}$$
$$a_2 = 100.5 \text{ hPa}^2 \text{ Mm}^{-2}$$

It has been shown so far that the pressure variance increases quadratically with distance, meaning that the pressure field is basically linear, which is a pre-condition for the application of Lindau’s method.

In the following, the coefficients $a_i$ are interpreted. The constant $a_0$ describes the effect of random observation errors. As two observations are involved, the value of $a_0$ has to be divided by 2 to obtain the error variance for a single observation. The square root of the error variance gives then the observation error in terms of standard deviation, given in hPa. Thus, the observation error $\varepsilon$ of pressure is:

$$\varepsilon = \sqrt{\frac{a_0^2}{2}} = 2.5 \text{ hPa}$$
The first order coefficient $a_1$ describes the linear growth of variance with increasing ship distance, introduced by stochastic variation of the pressure analogous to the Brownian motion. For a typical distance of 350 km between the ships, the stochastic contribution to the variance is 4.6 hPa$^2$, even less than the scatter induced by observation errors (12.4 hPa$^2$). The second-order coefficient describes the quadratic increase of the variance as a function of increased ship separation. This part of the variance is caused by large-scale linear differences, i.e. constant spatial slopes in the pressure field. At 350 km, 12.3 hPa$^2$ of the total variance can be ascribed to such large-scale pressure differences, which are used in Lindau’s method. This signal is considerably higher than the true stochastic variability, so that the assumption of a smooth linear pressure field is a good approximation. Above that, it can be shown that the magnitude of $a_2$ corresponds to a wind speed of about 8 m s$^{-1}$ (see Appendix). This shows that large-scale pressure differences are actually able to represent the observed wind. Consequently, large-scale pressure differences are a suitable means to calibrate the wind speed.

6. LINDAU’S METHOD

The method is based on the idea that neither triples of instantaneous pressure reports nor mean pressure gradients are suitable for a wind calibration. Instead, the absolute minimum of information, i.e. pairs of simultaneous pressure observations, is used. However, in this case the measured pressure difference not only depends on the wind speed but also on the orientation of the base line between the two ships within the pressure field. An estimate for this orientation is given by the wind direction, which is measured on board the ships. If the mean pressure differences, optionally converted into the geostrophic wind speed, are plotted as a function of the wind direction relative to the base line, the expected sine curve arises. Its phase shift determines the ageostrophic angle and its amplitude gives the magnitude of the mean geostrophic wind. Figure 8 gives an example of the result for the zonal band in the Atlantic between 40°N and 50°N in January for a specific decade.

We have to take into account the fact that any observation error will flatten the sine curve. If $\Delta d$ denotes the mean observation error, the amplitude of the obtained sine curve is diminished by a factor of $\cos \Delta d$, and that means the estimated geostrophic wind would be underestimated by that factor.
This effect is easy to realise. The sine curve is constructed from averages of different bins of the relative wind direction $d$. Some pressure differences may be sorted into the wrong bin owing to errors $\Delta d$ in the relative wind direction. The effect on the average is:

$$\sin(d + \Delta d) = \sin d \cos \Delta d + \cos d \sin \Delta d$$

(48)

$$= \sin d \cos \Delta d + \cos d \sin \Delta d$$

(49)

$$= \sin d \cos \Delta d$$

(50)

Equation (48) gives just the addition theorem of the sine function. The products under the overlines may be decomposed, as the errors are random (Equations (49) and (50)) and use the fact that the sine of an argument randomly fluctuating around zero is, on average, itself zero.

For proceeding further, the above-identified diminishing factor $\cos \Delta d$, caused by errors in wind direction, has to be determined. This is performed by a semivariogram using differences in the reported wind direction $D_1 - D_2$ between two simultaneously observing ships that are separated by a certain distance (Figure 9). A linear fit for $\cos(D_1 - D_2)$ as a function of ship separation allows us to extrapolate to zero distance, where only observation errors are responsible for values less than 1. Here, at zero distance, $D_1 - D_2$ becomes equal to $\cos^2 \Delta d$, since:

$$\cos(D_1 - D_2) = \cos(d_1 + \Delta d_1 - d_2 - \Delta d_2)$$

(51)

$$= \cos(\Delta d_1 - \Delta d_2)$$

(52)

$$= \cos(\Delta d_1) \cos(\Delta d_2) + \sin(\Delta d_1) \sin(\Delta d_2)$$

(53)

$$= \cos(\Delta d_1) \cos(\Delta d_2)$$

(54)

$$= \cos(\Delta d)^2$$

(55)

A short explanation of the above transformations is as follows: In Equation (51), the observed wind direction $D$ is decomposed into the true $d$ and the observing error $\Delta d$. For zero distance, the true wind directions are
equal (Equation (52)) and consequently cancel out. Equations (53) and (54) use again the addition theorem and the randomness of the errors. As the locations 1 and 2 are arbitrary, the mean errors are supposed to have the same magnitudes so that they can be reduced to \( \Delta d \) (Equation (55)).

Choosing again the example for January between 40°N and 50°N, the found zero distance value of \( \cos(\Delta d)^2 \) is 0.804, corresponding to an error in wind direction \( \Delta d \) of about 26° (Figure 9). These error effects are taken into account by dividing the amplitude of the fitted sine curve (Figure 9) by the square root of this factor, i.e. by \( \cos(\Delta d) = 0.897 \). For the specific zone, month and period considered here, the geostrophic wind is corrected from 13.3 to 14.8 m s\(^{-1}\), whereas the mean observed wind speed is 10.2 m s\(^{-1}\).

Figure 9. Determination of the mean observation error in the wind direction. The cosine of the difference in wind direction is plotted against the distance between the two observing ships. For zero distance, a measure of the errors, \( \cos^2 \Delta d \), is obtained.

Figure 10. The application of the method to different months and different latitudinal zones providing several pairs of geostrophic wind \( G \) and observed wind speed \( U \). A linear fit yields the \( G-U \) relationship for the reference period 1960 to 1971.
Then the procedure is repeated for all zones and months (Figure 10). A linear fit provides the relationship between geostrophic and observed wind speed ($G$ und $U$) for the specific period from 1960–1971. This period is used as reference because a corrected Beaufort scale has been derived just for that period (Lindau, 2002). In other years the $G - U$ relationship is shifted against the reference, which is considered to be spurious. A pair of correction factors and constants is needed to bring them into coincidence (Figure 11). In 1935, for instance, the factor is 0.856 and the constant 1.86 m s$^{-1}$, effecting an increase of the low and a decrease of high wind speeds in this specific year. We applied this method to every year and obtained yearly pairs of factors and constants, and thereby a time-dependent equivalent scale for a century from 1880 to 1980. Figure 12 depicts
Figure 13. Anomaly of the wind speed in COADS within the North Atlantic. The uncorrected time series shows significant trends since and prior to 1950, which are reversed or vanishing after the correction.

its major characteristics by six time series for the most frequent equivalent values of Beaufort 2 to 7. The striking features are the increased equivalent values for low Beaufort classes in the middle of the century. Using this scale, we are able to correct the time series of the raw wind reports of COADS. The originally found negative trend in historical times is reversed, and the positive trend afterwards vanishes nearly entirely (Figure 13).

7. CONCLUSIONS

The use of pressure differences for the calibration of marine wind data is undisputed because the data are completely independent and pressure observations belong to the standard ship reports so that the available data coverage is satisfactory. But the method to derive the pressure gradients is controversial. We reviewed three different approaches: The method of observation triples, the method of mean pressure differences and Lindau’s method using only two simultaneous pressure measurements plus the information of wind direction. The last is the only method that is able to provide reasonable results.

The method of observation triples suffers from huge error variance, which acts not only as random noise but additionally as a systematic and large offset. The pressure gradient measured by a triple of observations can be alternatively expressed as the variance between the three measurements. From this it is obvious that random observation errors have an additive effect and increase the result systematically. Since the errors do not remain constant through the years this method must fail.

By using the method of mean pressure differences, the observation errors are widely cancelled out. However, mean pressure differences are only proportional to the mean vector wind, but not to the scalar wind, which is intended to be corrected in most cases. The wind steadiness, defined as the ratio between scalar and vector winds, is shown to be not constant in time, which would be necessary for this method. Even decadal trends in both the vector and the scalar winds show only a weak correlation of less than 0.6, which is much too low for using this relationship as calibration.

As a prerequisite for Lindau’s method, a constant slope of the pressure field between the ships is assumed, since otherwise the measured pressure difference would not be representative for the actual wind. Using a
semivariogram, we verified that the pressure field exhibits actually a quadratic increase of variance, which is a proof for its basically linear character on this spatial scale.

Having clarified that the assumption of linear pressure fields is justified, Lindau's method is discussed and shown to perform better than the alternatives. It uses pairs of pressure differences, and averages the values for constant relative wind directions, which serve as estimates for the orientation of the base line between the two ships. During this averaging process the errors of the pressure observations are eliminated to a large extent, so that the difficulties inherent to the method of observation triples, discussed in Section 3, are overcome. But at the same time, the method is based on instantaneous pressure differences. Therefore, it is able to provide the intended measure for the scalar wind and not only a measure for the vector wind, as it is performed by the method of mean pressure gradient, discussed in Section 4. In return, errors in wind direction occur, which take effect in a diminished amplitude of the sine curve. But these effects are one order smaller than the signal and a reliable technique of correction is presented. Finally, Lindau’s method is applied to COADS wind data in the North Atlantic. The positive trend since the year 1950, found in the raw data, vanishes after the correction. The negative wind trend during the first half of the last century is reversed. The considerable impact of the correction shows that any simple interpretation of raw marine data has no firm basis.

APPENDIX

In this appendix we examine in detail the meaning of the second-order coefficient $a_2$, which is derived in Section 5 by fitting a quadratic function to the mean squared pressure differences between pairs of ships. The question arises whether the magnitude of $a_2$ reflects indeed the typical amount of large-scale pressure gradients. In other words: can the numerical value of $a_2$ be translated into the magnitude of the underlying mean gradient? It is possible, but the method is complex so that the discussion is separated from the main text.

According to Equation (46), the pure linear part of the pressure field, suppressing the effects of observation errors and stochastic variability, is described by:

$$\langle (\Delta p)^2 \rangle = a_2 (\Delta x)^2$$

(56)

For a fixed but arbitrary separation $\Delta x$ we can write:

$$a_2 = \left( \frac{\Delta p}{\Delta x} \right)^2$$

(57)

Thus $a_2$ represents the mean squared pressure gradient, which is of course not equal to the square of the mean gradient, so that the square root of $a_2$ does not actually provide the mean gradient we are searching for. However, this simple method yields, the correct result because the two effects cancel out each other.

The first effect is the following: The base line between the two ships is oriented arbitrarily in the pressure field. Depending on the angle $\beta$ between base line and isobars, only component $g$ of the total pressure gradient $g_0$ is measured by the ships. The procedure applied in Section 5 uses mean squared differences of $g$ thus:

$$\bar{g}^2 = g_0^2 \sin^2(\beta) = g_0^2 \sin^2(\beta)$$

(58)

The second equality is justified since the orientation of the base line is independent of the strength of the gradient. Assuming $\beta$ to be uniformly distributed, it follows:

$$\frac{1}{2\pi} \int_0^{2\pi} \sin^2 \beta d\beta = \frac{1}{2}$$

(59)

We can conclude that arbitrary orientated pairs observe on average only half of the true value, when the squared differences are considered.
This effect is cancelled out by the following: Consider an arbitrary sample consisting of \( n \) \( g_i \) with the variance \( V \), where bars denote the sample average:

\[
V = \frac{1}{n-1} (\Sigma g_i g_i - n \bar{g} \bar{g})
\]  

(60)

In Equation (57) we consider mean squared values, thus:

\[
\frac{1}{n} \sum g_i g_i = \frac{n-1}{n} V + \bar{g} \bar{g} \approx V + \bar{g} \bar{g}
\]  

(61)

Simply extracting the root mean squared values represents the mean of the sample only if the variance is zero. So, we need an estimate for the variance of pressure gradients. In this situation, it may be helpful to remember that the average and the standard deviation of the wind are of the same order of magnitude. We can assume that this is also true for the pressure gradient \( g \). In this case (and neglecting the factor \( (n - 1)/n \)) the squared mean is equal to half of the mean squared value:

\[
\bar{g} \bar{g} = \frac{1}{2} \bar{g}^2
\]  

(62)

Thus, extracting just the square root from \( a_2 \) overestimates the underlying gradient by a factor of \( \sqrt{2} \), as the variance of the gradient is not zero. On the other hand, the true strength of the gradient is underestimated by the same factor, since the base line between the two ships is not always oriented orthogonal to the isobars. We found \( a_2 \) to be 100.5 hPa\(^2\) Mm\(^{-2}\). Thus, an estimate for the underlying gradient is about 10 hPa per 1000 km. In the mid-latitudes such a gradient corresponds to about 8 m s\(^{-1}\), which is in perfect agreement with the observed wind speed. The linear part of the pressure field is obviously strong enough to be considered as an exclusive force of the observed wind.

REFERENCES


