Spatial kinetic energy spectra in the convection-permitting limited-area NWP model COSMO-DE

Lotte Bierdel∗, Petra Friederichs, Sabrina Bentzien

Lotte Bierdel
Meteorological Institute, University of Bonn
Auf dem Hügel 20, 53121 Bonn, Germany
phone: +49 228 735108
e-mail: lbierdel@uni-bonn.de
Abstract

Kinetic energy spectra derived from commercial aircraft observations of horizontal wind velocities exhibit a $k^{-5/3}$ wavenumber dependence on the mesoscale that merges into a $k^{-3}$ dependence on the macroscale. In this study, spectral analysis is applied to evaluate the mesoscale ensemble prediction system using the convection-permitting NWP model COSMO-DE (COSMO-DE-EPS). One-dimensional wavenumber spectra of the kinetic energy are derived from zonal and meridional wind velocities, as well as from vertical velocities. Besides a general evaluation, the model spectra reveal important information about spin-up effects and effective resolution.

The COSMO-DE-EPS well reproduces the spectral $k^{-5/3}$ dependence of the mesoscale horizontal kinetic energy spectrum. Due to the assimilation of high-resolution meteorological observations (mainly rain radar), there is no significant spin-up of the model simulations within the first few hours after initialization. COSMO-DE-EPS features an effective resolution of a factor of about 4 to 5 of the horizontal grid spacing. This is slightly higher in comparison to other limited area models. Kinetic energy spectra derived from vertical velocities exhibit a much flatter wavenumber dependence leading to relatively large spectral energy on smaller scales. This is in good agreement with similar models and also suggested by observations of temporal variance spectra of the vertical velocity.
Zusammenfassung


Das COSMO-DE-EPS reproduziert die \( k^{-5/3} \)-Abhängigkeit des Spektrums der kinetischen Energie auf der Mesoskala. Durch die Assimilation hochaufgelöster Radardaten zur Initialisierung des Modells gibt es keinen signifikanten Spin-Up während der ersten Modellstunden. Die effektive Auflösung des COSMO-DE-EPS beträgt das vier- bis fünffache des horizontalen Gitterabstands. Im Vergleich zu anderen Modellen mit begrenztem Modellgebiet ist die effektive Auflösung damit höher. Aus Vertikalgeschwindigkeiten abgeleitete Spektren der kinetischen Energie weisen eine flache Wellenlängenabhängigkeit auf. Dies führt zu einer relativ hohen spektralen Energie auf kleineren Skalen, was mit Beobachtungen zeitlicher Varianzspektren der Vertikalgeschwindigkeit übereinstimmt.
1 Introduction

The spectral variance of variables such as the kinetic energy as a function of horizontal scale is fundamental in many theoretical studies of geophysical fluids and in turbulence theory. A general observation when analyzing atmospheric horizontal kinetic energy spectra is a $k^{-3}$ slope on larger scales and a $k^{-5/3}$ slope on the mesoscale, in observational as well as model data (Larsen et al., 1982; Nastrom and Gage, 1985; Lindborg, 1999; Cho and Lindborg, 2001; Skamarock, 2004), where $k$ is the wavenumber.

Nastrom et al. (1984) analysed commercial aircraft measurements of wind velocity under the assumption of frozen turbulence, where a Taylor transformation is applied to convert frequency spectra into wavenumber spectra. In their study two possible interpretations of the empirically found $k^{-5/3}$ powerlaw dependence below scales of about 500 km are given: an inverse energy cascade (from high to low wavenumbers) in quasi two-dimensional (2D) turbulence or a forward energy cascade (from low to high wavenumbers) associated with a spectrum of internal gravity waves.

Since then, the theoretical interpretation of the mesoscale kinetic energy spectrum is under vital debate (Lindborg, 2005, 2007; Tulloch and Smith, 2009; Lovejoy et al., 2010). Mainly two hypothesis have been put forward. The first hypothesis (Gage, 1979; Gage and Nastrom, 1986) suggests an interpretation of the mesoscale kinetic energy spectrum in terms of geostrophic turbulence (Charney, 1971) which is in turn based on quasi 2D turbulence theory (Kraichnan, 1967). The energy spectrum of 2D-turbulence is characterised by both the conservation of energy and enstrophy. Fjørtoft (1953) and Charney (1971) based their analysis on triadic transfer in the wavenumber space. They obtain a $k^{-3}$ forward enstrophy cascade with zero energy flux and a $k^{-5/3}$ inverse energy cascade with zero enstrophy flux. It is pointed out by Merilees and Warn (1975) that the latter proof is flawed, and Gkioulekas and Tung (2007) give a more general proof considering net fluxes. They conclude that enstrophy
is predominantly transferred downscale and energy is transferred upscale in forced-dissipative
2D-turbulence under statistical equilibrium. Furthermore they emphasise that a downscale energy
transfer is not prohibited although it is shown that the net energy flux is directed from small to
large scales.

The interpretation of atmospheric motions in terms of 2D-turbulence theory has been ques-
tioned (Lindborg, 1999) mainly for two reasons: Firstly, 2D-turbulence theory requires a small-
scale source for the formation of an upscale energy flux on the mesoscales. However, this source
would be located on scales which are dominated by three-dimensional structures. Therefore, a
downscape energy cascade as predicted by 3D-turbulence theory should evolve rather than an
upscale cascade characteristic for 2D-turbulence. Secondly, there is observational evidence that
the $k^{-5/3}$ spectral slope associated to the energy cascade appears on smaller scales than the
$k^{-3}$ spectral slope of the enstrophy cascade. This is 'paradoxical' (Frisch (2004), p. 242) since
Kraichnan (1967) predicted the opposite in 2D-turbulence theory. The mesoscale portion of
the spectrum can still be explained as in 2D-turbulence by the formation of an inverse energy
cascade (Gage, 1979; Lilly, 1983; Metais et al., 1996) which requires a small-scale energy
source. Lilly (1983) suggested that this source can be generated by decaying convective clouds
and thunderstorm anvil outflows.

The second hypothesis bases on small wavenumber gravity waves which break down con-
tinuously to shorter waves providing a downscale (from large to small scales) energy flux (De-
wan, 1979, 1997; Van Zandt, 1982). The resulting forward cascade resembles the well-known
cascade of fully-developed, isotropic 3D-turbulence derived by Kolmogorov (see for example
Frisch (2004), p.72 ff.). Recent interest has focused on the interpretation of the $k^{-5/3}$ spec-
tral slope in terms of 3D stratified turbulence (Lindborg, 2005; Riley and Lindborg, 2008)
where further evidence for the forward cascade hypothesis is given. Lindborg (2006) gives an
analysis of highly stratified flows with Froude numbers much smaller than one. He argues that
two-dimensional vertically oriented vortices split into thin layers through the influence of strong stratification. These layers, in turn, split into thinner layers and so on. As the vertical extend of the layers becomes smaller, they show sensitivity to Kelvin-Helmholtz instabilities which cause a horizontal break-up of the layers. Through this mechanism total energy is transferred within a highly anisotropic forward cascade (Lindborg, 2005). Advocating the ‘wave-hypothesis’, Lindborg (2006) argues, that the word ‘wave’ should be replaced by ‘layer’ in order to avoid confusion with linear waves and to take account of the strong directional dependence of highly stratified motion simultaneously.

In addition to theoretical studies the observed spectral slope of the mesoscale portion of the spectrum is reproduced by direct numerical simulations (see for example Riley and De- Bruynkops (2003); Lindborg and Brethouwer (2007); Tung and Orlando (2002)) and numerical weather prediction (NWP) models. Koshyk and Hamilton (2001) performed general circulation model (GCM) simulations with the hydrostatic SKYHI GCM of the Geophysical Fluid Dynamics Laboratory with a horizontal grid spacing of 35 km of the highest resolution version. Their simulations reproduced both the $k^{-3}$ spectral slope on large scales of about 5000 km to 500 km and the $-5/3$ power-law dependence below 500 km. Skamarock (2004) computed kinetic energy spectra using NWP forecasts from the non-hydrostatic Weather Research and Forecast (WRF) model. He found that the $k^{-5/3}$-range is reproduced and actually develops in simulations which were initialized with large-scale fields lacking mesoscale variability.

The $k^{-5/3}$ slope on the mesoscale has significant implications for the predictability when using high-resolution mesoscale (NWP) models, since error growth on small scales may be much faster for mesoscale dynamics compared to synoptic scale motions (Skamarock, 2004). It is thus important to evaluate NWP models with respect to their spectral characteristics. In this study, spectral analysis is applied to evaluate the mesoscale ensemble prediction system (EPS) using
the convection-permitting NWP model COSMO-DE (Baldauf et al., 2011). Horizontal, one-
dimensional wavenumber spectra of the kinetic energy are derived from zonal and meridional
wind velocities as well as from vertical velocity. The spectra not only represent the spatial
characteristics of mesoscale fields in COSMO-DE, but also give information about the scale
of the dynamics that is effectively resolved by the model. Spin-up effects and the development
of mesoscale disturbances during the forecast as well as the effects of data assimilation are
investigated.

The spectral behavior of the kinetic energy components is assessed with special emphasis on
the following questions:

• Does COSMO-DE reproduce the $k^{-5/3}$ spectral slope for the horizontal velocity components
  on the mesoscale?

• Does the COSMO-DE ensemble prediction system (COSMO-DE-EPS) exhibit significant
  spin-up effects with respect to the spectral variance?

• What is the effective resolution of COSMO-DE with a grid spacing of 2.8 km?

• What are the spatial characteristics of the vertical velocity component?

Since COSMO-DE-EPS is aimed at forecasting high-impact weather conditions, we concen-
trate on cases that bear potential for hazardous impacts. The most dominant mesoscale weather-
related impacts in western Europe are related to strong mean winds, severe gusts, and heavy
precipitation (Craig et al., 2010). Particularly during summer, these weather situations are of-
ten related to deep moist convection.

Only very few observations exist of spatial variance spectra of vertical velocities on the atmo-
spheric mesoscale. Bacmeister et al. (1996) present horizontal wavenumber spectra of the ver-
tical velocity derived from stratospheric NASA aircraft campaigns. Time-dependent spectra have
been measured amongst others by radiosondes, aircrafts or mesosphere-stratosphere-troposphere
(MST) radar. Temporal variance spectra of horizontal velocities follow a $k^{-5/3}$ power law on temporal scales between 2 and 50 hours (LARSEN et al., 1982), whereas variance spectra of vertical velocities show a much flatter scale dependence (DAREN et al., 1987). Model spectra seem to reproduce the flatter spectrum for vertical wind velocities (SKAMAROCK and BRYAN, 2006; FIORI et al., 2010). However, SKAMAROCK and BRYAN (2006) point out that the spectral characteristics of the vertical velocity change with the grid spacing indicating that convergence of the numerical solution is not yet achieved.

Spectral decomposition of fields from limited-area models by simple Fourier transformation leads to largely biased estimates of the spectrum. Fourier transformation requires periodic boundary conditions. Otherwise, unresolved large scale changes alias to smaller scales and cause artificially high energy on small scales (ERRICO, 1985; DENIS et al., 2002). It is thus necessary to remove the aperiodic changes within the horizontal fields. In this study we follow ERRICO (1985) and remove a linear two-dimensional trend. We furthermore assume horizontal isotropy, and average the two-dimensional spectra over a torus defined by the length of the two-dimensional wavenumber vector. In order to reduce the sampling noise, the individual spectra from each of the 20 members of the EPS are averaged. The spread over the ensemble accounts for the uncertainty of the estimates.

In this article we proceed as follows: Section 2 shortly introduces the COSMO-DE model and the ensemble prediction system setup. The weather situation during the selected cases is represented in section 3. This is followed by a description of how the energy spectra are derived in section 4. In section 5 we investigate the horizontal and vertical energy spectra in the COSMO-DE-EPS for different forecast times and during four weather situations. The conclusions are given in Section 6.
2 Data

The Consortium for Small-scale Modeling (COSMO) model is a non-hydrostatic limited-area atmospheric model (www.cosmo-model.org) which is part of the operational model chain of the German Meteorological Service (Deutscher Wetterdienst, DWD). COSMO-DE is the high-resolution version with a horizontal grid spacing of about 2.8 km. It is nested into the COSMO-EU with a horizontal grid spacing of 7 km which in turn receives the lateral boundary conditions from the global NWP model GME. The model domain is centered over Germany and covers parts of its neighboring countries and the Alpine region with $421 \times 461$ grid points on a rotated regular longitude-latitude grid. The vertical discretization of the non-hydrostatic model uses 50 layers in orography-following $\sigma$-coordinates. The operational forecasts at DWD are issued for 21 hours and start every 3 hours.

On account of the relatively small horizontal grid spacing, COSMO-DE is able to explicitly simulate deep convection without parameterization (DOMS et al., 2005). An important feature of the operational COSMO-DE prediction system is the assimilation of surface precipitation rates derived from the German radar network using latent heat nudging (STEPHAN et al., 2008, LHN). A cloud microphysics scheme accounts for precipitation in form of graupel, snow and rain (REINHARDT and SEIFERT, 2006). Sub-grid scale turbulence is parameterized according to the level 2.5-model of MELLOR and YAMADA (1982) with a turbulence closure of order 1.5 (DOMS et al., 2005).

The $\sigma$-levels reach from 21500 m above ground down to 10 m. The dynamics on lower levels are highly affected by orography, whereas the upper few levels are strongly damped by Rayleigh damping (DOMS et al., 2005). Horizontal energy spectra of horizontal and vertical wind velocities are investigated on the $\sigma$-levels in order to avoid artificial signals from interpolation. As pointed out by SKAMAROCK (2004) computing spectra on constant pressure or height surfaces only leads to little significant differences in the results.
The present study investigates forecasts from an experimental version of the ensemble prediction system COSMO-DE-EPS, which is being developed at DWD. Under optimal conditions, COSMO-DE-EPS consists of 20 forecasts using the operational version of COSMO-DE. The different forecasts are driven by five perturbations of model physics and four different lateral boundary conditions. No perturbation of initial conditions is applied in this version of the EPS. Model physics are perturbed within the parameterizations of COSMO-DE by constant perturbations. The entrainment rate for shallow convection is changed from 0.0003 (default) to 0.002, the scaling factor for the thickness of the laminar boundary layer for heat from 1. (default) to 10 and 0.1, respectively, the critical value for normalized over-saturation from 4 (default) to 1.6, and the maximal turbulent length scale from 500 m (default) to 150 m. A description of these parameters and their default values is given in (SCHÄTTLER et al., 2009). The lateral boundary conditions are taken from four members of the COSMO-SREPS ensemble prediction system (MARSIGLI et al. (2008), ARPA-SIM, Bologna) which is a nested EPS with a grid spacing of 10 km. The four selected members of COSMO-SREPS a driven by the global models IFS, GME, GFS and UM.

The experimental 24h-forecasts were initialized daily at 00 UTC and performed for a period from May to October 2009. Technical problems as well as an unsteady data flow from COSMO-SREPS lead to large gaps within the data set of the experiment, particularly with regard to the number of available members. Only 35 days of the above mentioned period consist of the entire 20 member forecasts.

3 Weather situations

We investigate COSMO-DE-EPS forecasts for four days during the period from May to October 2009 that exhibit potentially high-impact weather conditions. During all days chosen, heavy precipitation occurred over parts of the model domain. All weather situations are prone for deep
convection, either within a synoptic scale frontal system or related to a large scale convectively
instable vertical stratification. Another criterion choosing these days is the availability of an EPS
with at least 15 members. The EPS is complete for May 20th, July 2nd, and August 21th 2009,
whereas for July 17th 2009 only 15 members are available.

During May 20th 2009, a cold front crosses the model domain from west to east. This leads to
a bisection of weather conditions over the model domain with a postfrontal north-western and a
prefrontal south-eastern region. The postfrontal part gets under the influence of cold polar air that
infiltrates in higher altitudes and leads to a destabilization of the atmosphere. The passage of the
cold front involves showery precipitation events in north-western Germany. During the course
of the day, the cold front becomes stationary and vanishes. The prefrontal part of the model
domain is dominated by subtropical, unstable air masses. After a clear morning, due to solar
irradiation cumulus clouds arise very locally in the south and south-east and lead to precipitation
and thunderstorms.

On July 2nd 2009, the weather condition over Germany is dominated by a high-pressure
system with weak horizontal pressure gradients. While it is fair in the morning, cumulus clouds
form over the central and sout-eastern part of the model domain in the late afternoon. The
development of intense and small-scale deep convection is a result of solar irradiation which
heats moist and warm air located over the model domain. Thunderstorms with hail and heavy
squalls arise locally, but no synoptic-scale lifting occurs during this day.

During July 17th 2009, a cold front of a low centered over England crosses the model domain
from south-west to north-east and infiltrates polar maritime air behind the front line. The front
is preceded by a convergence line with squall-line character lying in the warm-sector of the low,
in which course showers and thunderstorm cells form and intensify around noon. The synoptic-
scale induced storms are accompanied by hail and storm rainfall.

August 21th 2009 follows the hitherto warmest day of the year in Germany. It is characterized
by the passage of a cold front which is preceded by a convergence line. The prefrontal infiltration
of cold air leads to strong temperature differences between the eastern and western part of the
model domain. Along the convergence line thunderstorms and strong precipitation occur.

4 Methods

Generally, spectral analysis uses eigenfunctions of the Laplace operator within the respective
domain. The solutions of the two dimensional Laplace operator on a global domain are spherical
harmonics. On a limited area with regular grid spacing and periodic boundary conditions,
these solutions are sinusoidal functions. However, fields from limited-area NWP models have
non-periodic boundaries and are often formulated on a non-Cartesian horizontal grid. Spectral
analysis is thus not straightforward.

Fortunately, the horizontal grid of COSMO-DE is a rotated latitude-longitude grid where the
south pole is located at 40°S and 10°E. Consequently, the equator is located near the center of
the model domain, and the convergence of the meridians is minimized and the deviation from a
regular grid is negligible.

COSMO-DE-EPS is nested into members of coarser-scale COSMO-SREPS simulations (Sec.
2). The nesting uses a relaxation scheme. The influence of the lateral boundary condition
diminishes with distance to the boundary. However, the dynamics near the lateral boundaries
are strongly smoothed. For that reason, we removed 50 grid points (about 140 km) along each
edge of the model domain, so that it is reduced to 321 × 361 horizontal grid points.

Due to the lateral forcing, the boundary conditions are not periodic. If these aperiodicities are
not removed, the variance spectra are largely distorted due to the aliasing of the large, unresolved
changes onto smaller scales. In this study, they are removed by subtracting appropriate linear
trends following ERRICO (1985). Another methodology to account for the aperiodicities is the
discrete cosine transform (DCT, DENIS et al., 2002), which takes a mirror image of the original
field prior to the Fourier transformation, and therefore makes the analyzed field periodic.

The removal of trends will be shortly described in the following. We are given a field $a_{i,j}$ with $i = 1, \ldots, N_x$ and $j = 1, \ldots, N_y$ on the approximately regular reduced grid of COSMO-DE. The linear trend in $y$-direction is defined as

$$s_j = \frac{a_{N_x,j} - a_{1,j}}{N_x - 1},$$

and removed from $a_{i,j}$ by

$$a_{i,j}' = a_{i,j} - \frac{1}{2}(2i - N_x - 1) \cdot s_j.$$  \hfill (4.1)

Linear trend determination and subtraction are then repeated with interchanged $i$ and $j$. The resulting field is independent of the order of detrending in $x$- and $y$-direction and has periodic boundaries. Trend estimation and subtraction are separately performed for each model variable.

The estimation of the one-dimensional variance spectrum again follows Errico (1985). First, the two-dimensional spectrum is estimated via fast Fourier transformation (FFT). Assuming horizontal isotropy, the one-dimensional wavenumber spectrum $I(\kappa)$ is derived by summation of the squared absolute value of the two-dimensional Fourier coefficients $C(p,q)$ over a discrete annulus $A_\kappa$ in wavenumber space

$$I(\kappa) = \sum_{p,q \in A_\kappa} C_{p,q} \cdot C_{p,q}^*,$$

where $^*$ denotes the complex conjugate. $p$ and $q$ are the discrete wavenumbers in $x$ and $y$-direction and given by

$$p = \frac{2\pi l_x}{\Delta} \frac{1}{N_x - 1}, \quad l_x = 0, \pm 1, \ldots, \pm \frac{N_x}{2},$$

$$q = \frac{2\pi l_y}{\Delta} \frac{1}{N_y - 1}, \quad l_y = 0, \pm 1, \ldots, \pm \frac{N_y}{2},$$

where $\Delta$ is the horizontal grid spacing. $A_\kappa$ denotes the annulus given by $\kappa - \frac{1}{2} \Delta_\kappa < \sqrt{p^2 + q^2} < \kappa + \frac{1}{2} \Delta_\kappa$. Furthermore, $\Delta_\kappa$ is defined as the minimum of the fundamental values of $p$ and $q$, and
since in our case \(N_y > N_x\), we use

\[
\Delta \kappa = \frac{2\pi}{\Delta (N_y - 1)}
\]

to define the size of the annuli. The central radii of the annuli are defined as \(\kappa = l \cdot \Delta \kappa\) with \(l = 0, 1, ..., \frac{N_y}{2}\). For the kinetic energy spectrum of the horizontal wind, the one-dimensional spectra are independently estimated for the \(u\)- and \(v\)-wind component and then averaged.

In order to display the ensemble mean and spread of the spectra for the EPS simulations, we first calculated the spectra for the wind fields of each simulation, separately. The ensemble mean spectrum is then derived as the mean over the single spectra, whereas the spread indicates the range of the spectral estimates.

5 Results

5.1 Height-dependence of the mass specific, horizontal kinetic energy spectrum of the horizontal wind velocity

We start by investigating the height-dependence of the horizontal kinetic energy spectrum in the COSMO-DE-EPS. In the lower troposphere orography has a large impact on the spatial characteristics of the atmospheric dynamics, whereas the upper tropospheric flow is dominated by large scale structures. The upper most levels are strongly damped and exhibit very smooth horizontal wind fields. This is due to a damping layer, which is included in NWP models in order to absorb upward moving wave disturbances and prevent reflection of gravity waves from the upper model boundary (DOMS et al., 2005). In COSMO-DE the damping layer starts in an elevation of about 11 km (\(\sigma\)-levels < 10).

Fig. 1 illustrated the effect height-dependence of the horizontal kinetic energy spectra of the horizontal wind components. The spectra are presented for the 1h forecast time on May 20th 2009. The unphysical properties of the damping layer are reflected in the associated kinetic
energy spectra and are not considered (Figs. 1 a,b). The energy spectrum in $\sigma$-level 50 mainly
represents the influence of the orography with large energy particularly on smaller scales. Except
for the uppermost and the lowest levels, the $k^{-5/3}$ wavenumber dependence is well reproduced
throughout the entire troposphere. In order to minimize the influence of the upper damping layer
as well as the lower boundary, we concentrate on $\sigma$-level 20 for the investigation of the horizontal
kinetic energy spectra.

5.2 Mass specific, horizontal kinetic energy spectra of the horizontal wind velocity

In the following, spectra of the mass specific, horizontal kinetic energy for the four days described
in Sec. 3 are presented. Only $\sigma$-level 20 is considered here, which corresponds to a height of about
7 km.

Figs. 2 to 5 show energy spectra as well as the associated horizontal wind fields for the 1h,
10h, and 20h forecast times, respectively. The wind fields are composed of the absolute value of
the detrended fields of the wind components in m/s and the arrows which indicate the direction
and strength of the original fields of the wind components. We refrain from displaying the spectra
and wind fields for 0h forecast time (initialization). The variance spectra are almost identical to
those of 1h forecast time, except that as a result of the LHN (Sec. 2) the ensemble spread is
almost zero at initialization. The similarity of the spectra at 0h and 1h forecast times indicates
that no significant spin-up or adjustment takes place during the first forecast hours.

In Figs. 2 to 5 we display the ensemble mean of the energy spectrum as well as the spread
within the ensemble. Linear trends are fitted to the ensemble mean spectra on the log-log scale
over the range of the meso-$\beta$ scale (20 to 200 km) indicated by vertical gray lines. The slope
estimate $\beta$ is given in the caption together with a 95% confidence interval. Note however, that
the underlying regression model assumes independent Gaussian errors. Although most slope
estimates are very close to $k^{-5/3}$, there are significant differences.

We start with May 20th 2009. The horizontal wind in a height of about 7 km is dominated by a strong south-westerly component that weakens during the course of the day (right panels of Fig. 2). The weak maximum of wind speed in the east of the model domain indicates the anticyclonic arm of the jetstream. Due to LHN (Sec. 2), atmospheric fields in COSMO-DE-EPS already contain small scale disturbances at the initialization of the forecasts (not shown). Consequently, Fig. 2 a) exhibits an energy spectrum that is very close to $k^{-5/3}$ over the range of the meso-$\beta$ scale. The spectrum slightly subsides at scales below about 12 km. The general character of the horizontal kinetic energy spectra is conserved throughout the simulation and a spin-up effect in which small scale disturbances evolve is not detected. However, at forecast time 10h, the energy at wavelengths between 12 km to 30 km is significantly increased with respect to a $k^{-5/3}$ slope. It should be noted, that spectra at forecast times 10h and 20h feature an overall loss of energy on the mesoscale compared to the first hours of simulation. This is due to a weakening and northward shift of the maximum wind speed as indicated in the right panels of Fig. 2.

The ensemble spread due to perturbed model physics and different boundary conditions (compare Sec. 2) increases over time. In order to investigate the influence of the lateral boundary conditions and the perturbed model physics, the spectra of each group of simulation setup are compared. However, no systematic differences are found between the groups and all members equally contribute to the spread of the ensemble (not shown).

The formulation of dissipation, the grid size and the utilized integration scheme influence the models capability to resolve processes acting on scales near the resolution-limits of the model. Thus, the spectra reveal important information about the models ability to resolve small scale dynamics. The wavelength at which a model spectrum decays relative to a simulation with a finer grid spacing is referred to as the effective resolution (SKAMAROCK, 2004). Here, the effective resolution is roughly estimated as the wavelength where the spectrum decays significantly below
the $k^{-5/3}$ slope. The rationale behind this is that the $k^{-5/3}$ slope is expected to extend further down to smaller wavelengths. If the spectrum drops more rapidly then those modes are damped by numeric dissipation and hence are dynamically inaccurate.

With regard to the spectra on May 20th 2009 (Fig. 2), the effective resolution is estimated to about 15 km since on smaller scales the spectral slope decays significantly faster than $k^{-5/3}$.

SKAMAROCK (2004) investigates the effective resolution of the WRF model with different grid spacing. He observes that the effective resolution is a constant multiple of the horizontal grid spacing $\Delta$, which is generally around $7\Delta$. In our case, an effective resolution of 15 km corresponds to $5\Delta$, which is higher than in SKAMAROCK (2004).

On July 2nd 2009, as described previously (Sec. 3), the model domain is under the influence of a high pressure system and small-scale convection arises during the course of the day. Consistently, the absolute values of the horizontal wind components (right panel of Fig. 3) are very low and large scale dynamics are less pronounced. In contrast to May 20th 2009, no jetstream passes the model domain. The $k^{-5/3}$ slope on the mesoscale is again well reproduced (Fig. 3) and the ensemble spread evolves similarly to that on May 20th. One major difference is the less pronounced decay of the spectrum at higher wavenumbers indicating a relatively high portion of small-scale variations of the horizontal kinetic energy. This is consistent with a weather situation where the dynamics are dominated by local thermal convection rather than synoptic-scale processes. The weak transition from resolved to unresolved scales induces difficulties in determining the effective resolution. An approximate estimation yields an effective resolution of about 11 km corresponding to $4\Delta$. At forecast hour 20, a significant increase is observed with respect to a $k^{-5/3}$ slope in the kinetic energy on scales below 30 km.

Fig. 4 represents the kinetic energy spectra and the wind fields for July 17th 2009. The temporal evolution of the wind field is characterized by a jetstream entering the model domain. Again, the observed $k^{-5/3}$ slope is reproduced and maintained during the simulation. Compared
to the other spectra, the decay at the effective resolution of about 15 km is more pronounced. The
ensemble spread grows faster than during the previously discussed days. At forecasts hour 10 the
jetstream is located in the model domain. This coincides with an increase of small-scale energy
at the forecast hours 10 and 20, which in turn correlates well with the associated horizontal wind
fields, where small-scale variability enhances across the entire model domain. A striking feature
in Fig. 4 is the bulge across the wavelength range of about 120 km to 250 km in the spectrum
of the 20th forecast hour. This local increase of energy over a narrow range of wavelengths
accompanied by an enlarged ensemble spread probably represents the jetstream.

The horizontal kinetic energy spectra and the wind fields for August 21th 2009 are displayed
in Fig. 5. The temporal evolution of the horizontal wind field shows the weakening and north-
eastward displacement of the cyclonic arm of the jetstream. Again, the $k^{-5/3}$ slope is well
reproduced, and the strong decay of the spectra at small scales reveals an effective resolution of
about 13 km. As in the previous cases the ensemble spread grows with time and some enhanced
small scale energy is observed at forecast hour 20.

Figs. 2 to 5 indicate the spread within the ensemble in terms of the spectral kinetic energy.
It represents the spread in an average over the squared amplitudes of the respective harmonics.
Note, that the spectra are largely smoothed, firstly by averaging over all coefficients with similar
absolute value of the wavenumber vector, and secondly by averaging over the meridional and
zonal wind components. Another component of the ensemble spread is related to the phase shift,
which is associated to the displacement errors in an ensemble and is not investigated in this study.
Consequently, Figs. 2 to 5 only represent one smoothed component of the spread in the forecast,
and further studies are needed in order to assess the spread of the forecasts of the horizontal
kinetic energy spectra in the COSMO-DE-EPS with respect to observations.

Preliminary investigations of precipitation, surface temperature and wind gusts show that
COSMO-DE-EPS is underdispersive. This underdispersiveness is also observed in the AROME
mesoscale EPS from Météo-France (Vité et al., 2011). Note that the EPS we investigate here is started from unperturbed initial conditions. A perturbation of the initial conditions increases the spread during the first 6 forecasts hours (PERALTA and BUCHHOLD, 2011). However, the ensemble remains underdispersive (personal communication S. Theis, 2011).

5.3 Mass specific, horizontal kinetic energy spectra of the vertical wind velocity

This section discusses the variance spectra of the mass specific vertical velocity (Fig. 6) on $\sigma$-level 20. We restrict the presentation to the 20h forecast time of the four days described in Sec. 3. The computation is described in Sec. 4. The spectral behavior of the vertical velocity exhibits a much flatter wavenumber dependence compared to the horizontal velocity components. A $k^{-5/3}$ dependence, and therefore an inertial subrange, is not existent. In contrast, all the spectra show almost no wavenumber dependence on the mesoscale.

This is consistent with horizontal power spectra of the stratospheric vertical velocities derived from aircraft observations by BACMEISTER et al. (1996). Similarly to the simulated spectra, their spectral slope is flat on wavelengths above about 15 km, and drops rapidly towards small wavelengths below 15 km. The flat shape of the spectrum is also observed in model studies by SKAMAROCK and BRYAN (2006) and Fiori et al. (2010), and consistent with MST-observations of temporal variance spectra of the vertical velocity by LARSEN et al. (1982) and DAREN et al. (1987). The spectral characteristics of the horizontal and vertical components of the kinetic energy are very different. The large scale variations are dominated by the horizontal kinetic energy component, whereas the contribution of the vertical component becomes significant for small-scale variations.

The general characteristics of the vertical velocity spectra for the other forecasts times are very similar, and therefore not displayed. As for the horizontal energy spectra, the ensemble
spread is very small during initialization, but the ensemble spread grows faster for the vertical 
compared to the horizontal velocity components (not shown).

The decay of the spectra at very high wavenumbers strongly varies between the four days. On 
July 2nd 2009 (Fig. 6 b)) the damping of the spectrum is much less pronounced than on the other 
days. A similar behavior was indeed observed on July 2nd 2009 for the horizontal velocity spectra 
in Figs. 2 to 5. The estimation of an effective resolution on the basis of vertical velocity spectra 
is critical since no hypothesis exists about the behavior on smaller scales. However, the spectra 
drop significantly at wavelengths of about $4\Delta$ to $5\Delta$ probably due to numerical dissipation. This 
is consistent with the estimates of the effective resolution for the horizontal wind velocities.

6 Conclusions

Mass specific horizontal kinetic energy spectra have been computed from horizontal fields of the 
horizontal and vertical wind velocity as simulated by the COSMO-DE-EPS. The energy spectra 
are estimated following Errico (1985). The aim of the study is to assess the spectral behavior of 
the kinetic energy components in COSMO-DE, i.e. whether COSMO-DE reproduces the $k^{-5/3}$ 
spectral slope for the horizontal velocity components on the mesoscale. Furthermore, we assess 
the effective resolution of COSMO-DE with a grid spacing of 2.8 km and investigate the spatial 
characteristics of the vertical velocity component.

To that end, 21 hour forecasts for four days in Summer 2009 are investigated during weather 
situations with a potential for high-impact weather. We discuss the height dependence of the 
spectral behaviour and variance spectra for velocities in the upper troposphere are presented. 
Despite some significant deviations, the $k^{-5/3}$ slope characteristic of the mesoscale as suggested 
by flight observations by NASTROM and GAGE (1985) is generally reproduced in the COSMO-
DE-EPS. Albeit significant, the deviations are small and represent specific weather conditions
such as squall-lines or small convective cells. A consolidated view gives confidence in the high-resolution COSMO-DE in the sense that its dynamical representation is energetically correct.

Observations of the horizontal variance spectra of the vertical velocity are rare. The spectra of the vertical velocity component of COSMO-DE are very flat and show almost no scale dependence on the mesoscale. This is not inconsistent with observations of temporal variance spectra (Larsen et al., 1982; Daren et al., 1987) and confirms the findings by Fiori et al. (2010). It’s to be assumed, that the convergence of the numerical solution with respect to the spectral characteristics of the vertical velocity component is not yet achieved, as pointed out by Skamarock and Bryan (2006). Moreover, the spatial characteristics of the vertical velocity might strongly depend on the respective turbulence parameterization, and further studies are needed to assess the reasons of the flat vertical velocity spectra and how representative they are.

In all cases, the ensemble spread is very small at initialization. The horizontal wind fields exhibit a $k^{-5/3}$ energy spectrum already at initialization, and no spin-up effect or adjustment is observed. This is attributable to the LHN which assimilates high-resolution radar observations. With increasing forecast time the ensemble spread grows. The error growth as represented by an increased spread of the energy spectra depends on the weather situation. It is particularly large on July 17th 2009, when the jetstream enters the model domain. Similar results are obtained for the vertical velocity component. No spin-up is detected, but the error growth within the ensemble is larger.

The effective resolution of COSMO-DE is about 11 km to 15 km which corresponds to a multiple of 4 to 5 of the horizontal grid spacing. Skamarock (2004) concluded, that with an effective resolution of $7\Delta$ they are close to the limit of the resolving capabilities of finite-difference models that use explicit methods. This suggests, that the tuning of the dissipation or the formulation of the integration scheme in COSMO-DE optimizes the dynamical resolution of the model.
This study did not examine spectra with respect to the different theories described in the introduction. In order to confront COSMO-DE with theoretical aspects a range of hypothesis related to the different theoretical models for the $k^{-5/3}$ spectral slope on the mesoscales might be tested. Much insight could be gained for example through the analysis of third order structure functions and the division of the spectra into divergent and rotational modes.

Acknowledgments

The authors would like to thank the German Meteorological Service (DWD) for kindly providing the COSMO-DE ensemble simulations, and Susanne Theis (DWD) for her support. Special thanks are due to Andreas Hense, Andreas Bott, Matthieu Mabou, Werner Schneider and the two reviewers for their thoughtful comments on the manuscript.
List of Figures

1 Kinetic energy spectra of horizontal wind velocities for August 21th 2009 and 1h forecast time on σ-level a) 1 in 21.500 m, b) 10 in 13.620 m, c) 20 in 7.280 m, d) 30 in 3.136 m, e) 40 in 832 m, and f) 50 in 10 m. The thin line indicates the ensemble mean of the spectra, and the filled area is limited by the range of the ensemble spread. The dashed line corresponds to a $-5/3$ slope. The wavenumber is given in rad/m (lower x-axis) and the wavelength in km (upper x-axis).

2 Kinetic energy spectra and wind fields in about 7000 m for May 20th 2009 for a) 1h, b) 10h, and c) 20h forecast time are displayed. The thin line indicates the ensemble mean of the spectra (left panels), and the filled area is limited by the range of the ensemble spread. The dashed line corresponds to a $-5/3$ slope. The slope coefficients as displayed by dotted lines are estimated over the spectral range between the vertical gray lines. They amount to about a) $-1.78 \pm 0.1$, b) $-1.59 \pm 0.12$, and c) $-1.78 \pm 0.09$, respectively. The shading of the corresponding horizontal wind components (right panels) represent the absolute value of the detrended fields in m/s, and the arrows indicate the wind of the original fields.

3 Same as Fig. 2 but for July 2nd 2009 with the slope coefficients a) $-1.78 \pm 0.1$, b) $-1.84 \pm 0.1$, and c) $-1.25 \pm 0.1$.

4 Same as Fig. 2 but for July 17th 2009 (15 member ensemble) with slope coefficients a) $-1.58 \pm 0.1$, b) $-1.67 \pm 0.07$, and c) $-1.73 \pm 0.08$.

5 Same as Fig. 2 but for August 21th 2009 with slope coefficients a) $-1.46 \pm 0.08$, b) $-1.55 \pm 0.08$, and c) $-1.46 \pm 0.11$. 

23
Kinetic energy spectra of the vertical velocity in about 7000 m for forecast time 20h for a) May 20th 2009, b) July 2nd 2009, c) July 17th 2009, and d) August 21th 2009. The thin line indicates the ensemble mean. The filled area is limited by the range of the ensemble spread.
References


28


Figure 1: Kinetic energy spectra of horizontal wind velocities for August 21th 2009 and 1h forecast on $\sigma$-level a) 1 in 21.500 m, b) 10 in 13.620 m, c) 20 in 7.280 m, d) 30 in 3.136 m, e) 40 in 832 m, and f) 50 in 10 m. The thin line indicates the ensemble mean of the spectra, and the filled area is limited by the range of the ensemble spread. The dashed line corresponds to a $-5/3$ slope. The wavenumber is given in rad/m (lower x-axis) and the wavelength in km (upper x-axis).
**Figure 2:** Kinetic energy spectra and wind fields in about 7000 m for May 20th 2009 for a) 1h, b) 10h, and c) 20h forecast time are displayed. The thin line indicates the ensemble mean of the spectra (left panels), and the filled area is limited by the range of the ensemble spread. The dashed line corresponds to a $-5/3$ slope. The slope coefficients as displayed by dotted lines are estimated over the spectral range between the vertical gray lines. They amount to about a) $-1.78 \pm 0.1$, b) $-1.59 \pm 0.12$, and c) $-1.78 \pm 0.09$, respectively. The shading of the corresponding horizontal wind components (right panels) represent the absolute value of the detrended fields in m/s, and the arrows indicate the wind of the original fields.
Figure 3: Same as Fig. 2 but for July 2nd 2009 with the slope coefficients a) $-1.78 \pm 0.1$, b) $-1.84 \pm 0.1$, and c) $-1.25 \pm 0.1$. 
Figure 4: Same as Fig. 2 but for July 17th 2009 (15 member ensemble) with slope coefficients a) $-1.58 \pm 0.1$, b) $-1.67 \pm 0.07$, and c) $-1.73 \pm 0.08$. 
Figure 5: Same as Fig. 2 but for August 21th 2009 with slope coefficients a) $-1.46 \pm 0.08$, b) $-1.55 \pm 0.08$, and c) $-1.46 \pm 0.11$. 

34
Figure 6: Kinetic energy spectra of the vertical velocity in about 7000 m for forecast time 20h for a) May 20th 2009, b) July 2nd 2009, c) July 17th 2009, and d) August 21th 2009. The thin line indicates the ensemble mean. The filled area is limited by the range of the ensemble spread.