Paleo-climatic time series: statistics and dynamics

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Abstract. Simple models of the earth’s energy balance are instrumental for interpreting some qualitative aspects of the dynamics of paleo-climatic data. In the 1980s this led to the investigation of periodically forced dynamical systems of the reaction-diffusion type with small Gaussian noise, and a rough explanation of glacial cycles by Gaussian metastability. A spectral analysis of Greenland ice time series performed at the end of the 1990s representing average temperatures during the last ice age suggest an \( \alpha \)-stable noise component with an \( \alpha \sim 1.75 \). Based on this observation, papers in the physics literature attempted an interpretation featuring dynamical systems perturbed by small Lévy noise.

In terms of statistics of stochastic processes, this leads to a model selection problem. For instance, if the time series is modeled as a dynamical system perturbed by \( \alpha \)-stable noise, one needs an efficient testing method for the best fitting \( \alpha \). We develop a statistical testing method based on the \( p \)-variation of the solution trajectories of SDE with Lévy noise, for example by showing asymptotic normality or asymptotic \( \beta \)-stability of their approximations along finite interval partitions.

Generalizing the solution of the model selection problem, we are led to a class of reaction-diffusion equations with additive \( \alpha \)-stable Lévy noise, a stochastic perturbation of the Chafee-Infante equation. We study exit and transition between meta-stable states of their solutions. Due to the heavy-tail nature of an \( \alpha \)-stable noise component, the results differ strongly from the well known case of purely Gaussian perturbations. As opposed to the Gaussian picture, where a potential division has to climb to the nearest saddle of the underlying potential landscape, \( \alpha \)-stable exits occur as big enough jumps of the noise process.