Robust trend estimation: method and application to observational German precipitation data

Silke Trömel[*], Christian-D. Schönwiese

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[*]Corresponding author: Silke Trömel, Meteorological Institute of the University of Bonn, Auf dem Hügel 20, D-53121 Bonn, Germany, e-mail: silke.troemel@uni-bonn.de

Abstract. Trends in climate time series are habitually estimated on the basis of the least-squares method. This approach is optimal if the residuals are Gaussian distributed, but only a small number of observed climate time series fulfill this assumption. Here we introduce a new and robust method of trend estimation independent from Gaussian assumptions. This new method can be seen in the context of Trömel et al. (2005) who applied a generalized time series decomposition technique to monthly precipitation sums from a German station network of 132 time series covering 1901-2000 in order to achieve a statistical modeling of the time series. The time series under consideration can be interpreted as a realization of a Gumbel-distributed random variable with time-dependent scale and location parameter. More precisely, each observed value can be seen as one possible realization of the estimated probability density function (PDF) with the location and the scale parameter of the respective time step. Consequently, the expected value of the Gumbel distributed random variable can be estimated for every time step of the observation period and the statistical modeling represents an alternative approach to estimate trends in observational precipitation time series. The influence of relatively high precipitation sums is not greater than justified from the statistical point of view and changes in different parameters (here location and scale parameter) of the distribution can be taken into account. Monte-Carlo-simulations demonstrate the smaller mean squared error of the the trend estimator using the statistical modeling. The least-squares estimator often shows a positive bias, while the method introduced provides robust monthly trend estimates taken into account the statistical characteristics of precipitation.

1 Introduction

One important task of statistical climatology is the description of climate variability. Observed climate time series represent a reliable basis for statistical analyses. The simplest and broadly used model in trend analyses is the interpretation of the time series as a superposition of a linear trend and Gaussian noise, i.e. the deterministic part is restricted to a trend and the residuals should follow the Gaussian distribution. However, this simple model is insufficient to achieve a complete description of most of the observed time series. So, Grieser et al. (2002) consider temperature time series as a superposition of trends, annual cycle, episodic components, extreme events and noise. In that case, the residuals can not be distinguished from the realization of a Gaussian distributed random variable. The mean value at a given time is determined easily as the sum of the detected analytical functions. The so called distance function used to fit the analytical function is the quadratic function and the associated method including the minimization rule of quadratic deviations from fitted functions is the popular least-squares method. The estimator becomes a maximum likelihood estimator if the residuals can not be distinguised from the realization of a Gaussian random variable. Obviously, under the assumption of Gaussian-distributed residuals, the more deviant the point the greater the weight. The influence increases very fast. The expected probability of occurence is very small, so great relevance is attached to their occurence. Figure 1 illustrates the sensitivity of the least-squares method to relatively high values. On the left hand side, all values are in line with a negative linear trend. On

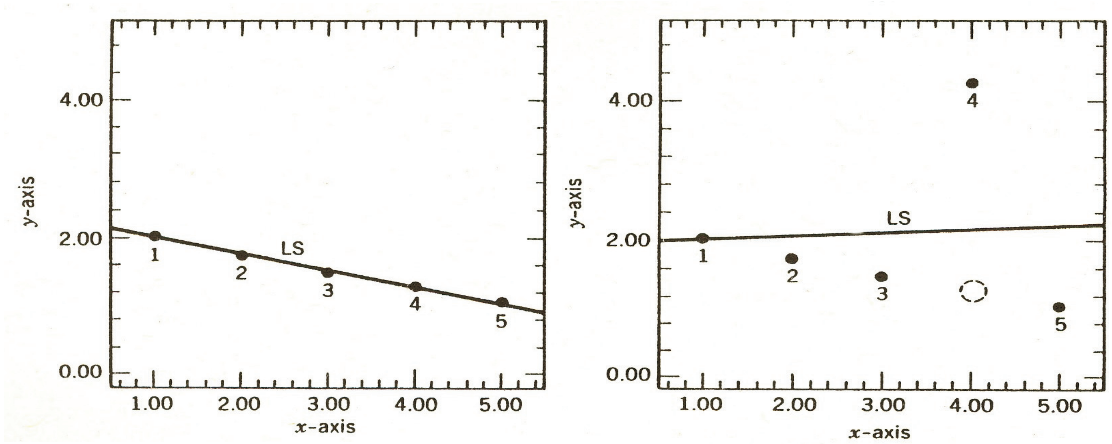

Figure 1: Sensitivity of the least-squares estimator to extreme values (Rousseeuw, 1987).

the right hand side, it is just one single value turning the negative into a postive signed trend. The influence of a single value is unlimited, i.e.the greater the value considered in Figure 1 the greater the amplitude of the estimated trend.

One simple step to robust trends is the elimination of extreme values before trend analyses are performed. A similar approach is presented by Huber (1981). The main idea of Huber-k estimators is to prevent the quadratic influence of values more deviant than k units from the mean value.

Huber-k estimators are less sensitive to extreme values but it is also possible to take into account the actual distribution of monthly precipitation time series. The tails are more pronounced and a greater number of relatively high deviations can be expected. Contrary

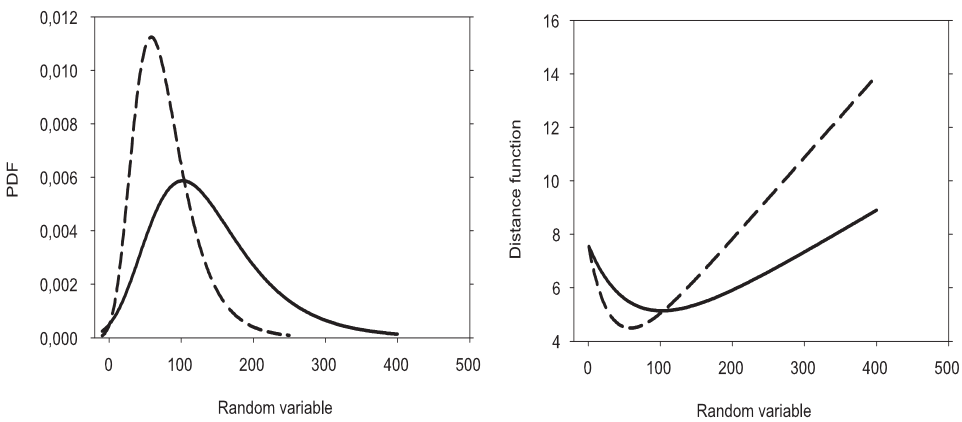

Figure 2: The PDF (left) of the Gumbel distribution and the distance function (right).

to the Gaussian distribution, observed precipitation distributions are skewed to the left. Furthermore, the Gaussian model is not suitable to describe observed changes in the variance of precipitation time series. Therefore, another statistical model should be applied for description of precipitation time series. Consistently with the maximum likelihood principle another distance function, defined as the negative logarithm of the PDF, replaces the function of squared errors to be minimized, if we choose another distribution as basis of the decomposition procedure. Additionally, time dependence for different distribution parameters can be allowed. With the exchange of the distance function, fitted basis functions describe changes in the location, scale or shape parameter of an appropriate PDF. As an example, the corresponding distance function of the Gumbel distribution is

$$\rho(x,t) = \ln(b(t)) + exp\left(-\frac{x-a(t)}{b(t)}\right) + \frac{x-a(t)}{b(t)}. \tag{1}$$

Application of a generalized time series decomposition technique (Trömel et al., 2005 and 2006) provides a full analytical description of the observed monthly precipitation time series. The analytical functions mentioned above are now used to describe the scale and the location parameter of the Gumbel distribution. Figure 2 shows on the left hand side the PDF of the Gumbel distribution for two different location and scale parameters. On the right hand side the respective distance functions can be seen. The tails are more prominent and consequently, if we take a look at the distance functions, the influence increases less rapidly than in the quadratic case. We also can see that one value in a given distance from the location estimator has more weight the smaller the scale parameter. In this way, structured components can be detected in different parameters and estimators of different parameters compete with each other.

It is worth mentioning in this context that the Gumbel model is not sufficient for all monthly precipitation time series. Already within the European precipitation regime the shape of the distribution often shows an annual cycle or may reveal long-term trends. A statistical modeling based on the Weibull distribution represents an adequate description in these cases (see again Trömel et al., 2005). The Weibull distribution owns three parameters, namely the location, the scale and the shape parameter. Not until arid or semi-arid precipitation regimes are considered the method fails. In every month of the year a sufficient amount of rain is required to estimate a PDF for every time step of the observation period.

In Section 2 of this paper the definitions of the expected value of a Gumbel distributed random variable and a Weibull distributed random variable are given. These equations can be used to create trend maps for precipitation. Contrary to the well known linear trend maps on the basis of the least-squares estimator, this alternative approach takes into account the skewness of the distribution and changes in different parameters of the distribution. In Section 3 some results of an application to a German station network of 132 precipitation time series are presented. A subsequent presentation of results of performed Monte-Carlo simulations underlines the advantages of the trend estimator introduced and quantifies the mean squared error of both, the least-squares estimator and the estimator based on the statistical modeling of the time series. The appendix comprises the trend maps of the remaining months which are not included in Section 3.

2 Method

The expected value $\mu(t)$ at time t of a Gumbel-distributed random variable is defined as

$$\mu(t) = a(t) + \gamma b(t) \tag{2}$$

(Rinne, 1997) with Eulers constant $\gamma = 0.57722$, indicating that changes in the scale a(t) and the location parameter b(t) reveal changes in the mean value. So, if the decomposition procedure succeeds, the full analytical description of monthly precipitation series provides the expected value for every time step of the observation period. This result can be used to take into account the skewness of the distribution and changes in different parameters to estimate trends in the mean value.
Adressing N years of monthly climate time series, the trend of a Gumbel-distributed climate variable in a specific month j=1,...,12 can be defined as

$$\Delta\mu_j = a(T_{Nj}) + \gamma b(T_{Nj}) - a(T_{1j}) - \gamma b(T_{1j}) \tag{3}$$

and the new time variable T_{ij} with subscripts i=1,...,N and j=1,...12 denotes the jth month in the ith year of the observation period.

The expected value $\mu(t)$ at time t of a Weibull-distributed random variable with constant location parameter a_W=0 mm and time-dependent scale parameter b_W(t) and shape parameter c_W(t) is defined as

$$\mu(t) = b_W(t)\Gamma\left(1 + \frac{1}{c_W(t)}\right) \tag{4}$$

(Rinne, 1997) with the Gamma-function

$$\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt. \tag{5}$$

Analogous to Eq. 3 the trend in a specific month j=1,...,12 of a Weibull-distributed climate variable can be defined as

$$\Delta\mu_j = b_W(T_{Nj})\Gamma\left(1 + \frac{1}{c_W(T_{Nj})}\right) - b_W(T_{1j})\Gamma\left(1 + \frac{1}{c_W(T_{1j})}\right). \tag{6}$$

The new approach includes several advantages. If the trend in the expected value is estimated on the basis of the method introduced, the sample size includes all N monthly precipitation sums. Contrary, the least-squares estimator only takes N/12 values into account in order to estimate the trend of a specific month during an observation period of N years. The trend observed in January for example is estimated on the basis of all monthly precipitation sums observed in January.

Furthermore, changes in the expected value are caused by changes in the parameters of the distribution. Temporal changes in these parameters again, are given as the superpostion of significant detected trends, seasonal variations and the episodic component. These components are estimated on the basis of the whole time series. Consequently, estimated trends in different months are less sensitive to single values. Seasonal differences concerning the sign and the magnitude of the trend are given by long-term changes in the annual cycle of the distribution parameters and further several significant changes in the parameters. Abrupt rises of the trend estimates from one month to another caused by single relatively high values are suppressed. These values may occur by chance and are not attributed to a trend. A further contribution to monthly robust trend estimates is the fact that detected extreme events are extracted and do not influence the trend.

We already mentioned that the generalized decomposition technique avoids false statistical assumptions, too. Under Gaussian assumptions with constant variance, the quadratic function is used to fit the linear trend for example. The use of the statistical justified distance function ensures the correct weighting of the observed values in order to estimate structured components in the distribution parameters. The tails of the Gumbel or the Weibull distribution are asymptotically much larger than any corresponding Gaussian. Relatively high values get less influence, because their occurence is not as unlikely as in the Gaussian case.

So, that the method introduced provides robust monthly trend estimates, which take into account the non-Gaussian characteristics of observed precipitation time series.

3 Application to observational German precipitation data

In this chapter the application of the generalized decomposition technique to estimate changes in the expected value of precipiation time series, that is ordinary trends, is discussed. Figure 3 shows trend estimates for January (left chart) on the statistical modelling method introduced in this paper compared with the results of linear trend estimates on the basis of ordinary least-squares method (right chart). Obviously, in January both estimators provide simular spatial structures. We observe positive trends in the western and the southern part of Germany and negative or nearly unchanged mean values in the eastern part of Germany. Consequently, these results confirm the expected result, that the use of the least-squares estimator for non-Gaussian precipitation time series often impliessb a positive bias. Because of the greater tail of the Gumbel distribution, Gaussian assumptions often generate or amplify a trend. While a stationary Gumbel distribution could still explain the occurence of some relatively high values for example near the end of observation, Gaussian assumptions cause a shift of the distribution in order to explain their occurence. The comparison of the distance function of the Gaussian distribution with the distance function of the Gumbel distribution illustrates the difference between the estimators. But this is not the only difference. Figure 4 shows again both trend maps but for August. Now, the differences are more conspicuous. The least-squares estimator assumes negative trends in the greatest part of Germany. Compared to January, the negative trends in the eastern part of Germany are now more pronounced, even if the Gumbel model is used. However, the trend map still shows postive trends in the south. In the statistical modelling of the whole time series based on the Gumbel model no significant change in the sign of the trend is detected. The least-squares estimator is applied to 12 time series separately in order to yield the trend for every single month. Together with the different distance function, this approach is less robust and generates greater differences in the trends of the single months. Contrary, statistical modelling is smoothing the seasonal trend estimates in the mean value. Great differences in trends of subsequent months are condemned as untrustworthy from a meteorological point of view. The reader should also compare monthly trend maps of subsequent months on the basis of the Gumbel model (see Trömel (2004) for the other trend maps of the year) with trend maps estimated on the basis of the least-squares method (Rapp, 1996).

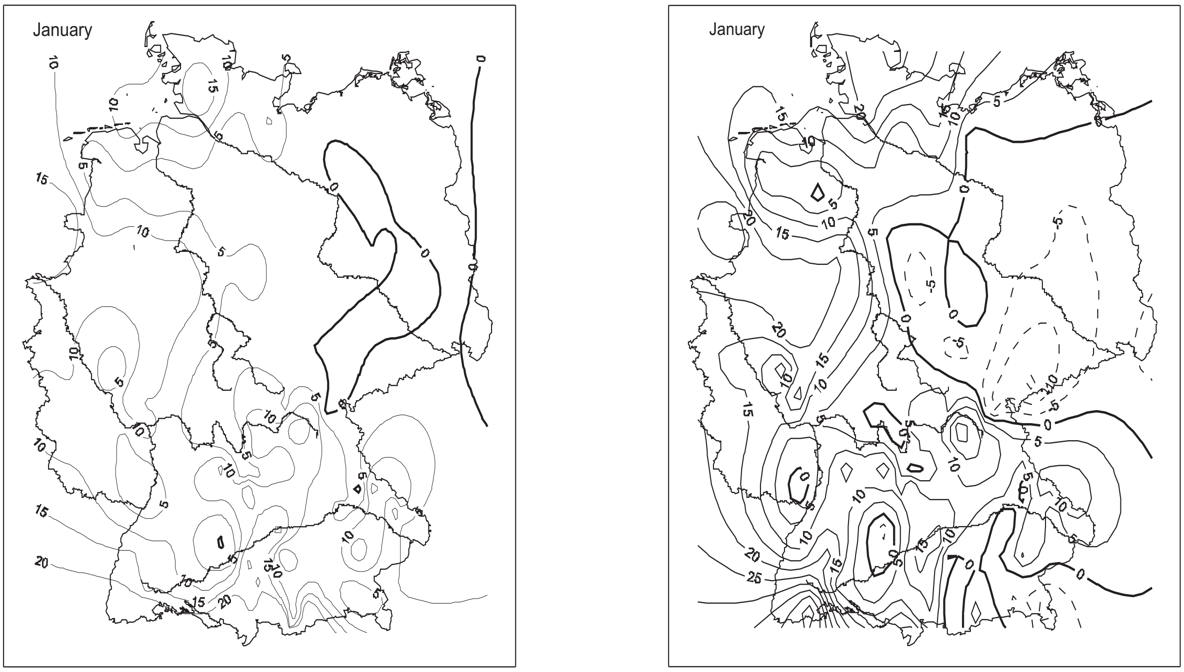

Figure 3: Trends in the expected value in January estimated on the basis of the statistical modeling (left) compared to the linear trend estimates using Gaussian assumptions.

3.1 Monte Carlo simulations

In the following, the results of Monte-Carlo simulations should clarify whether the more extensive statistical modeling really provides the more reliable trend estimates. In partic-

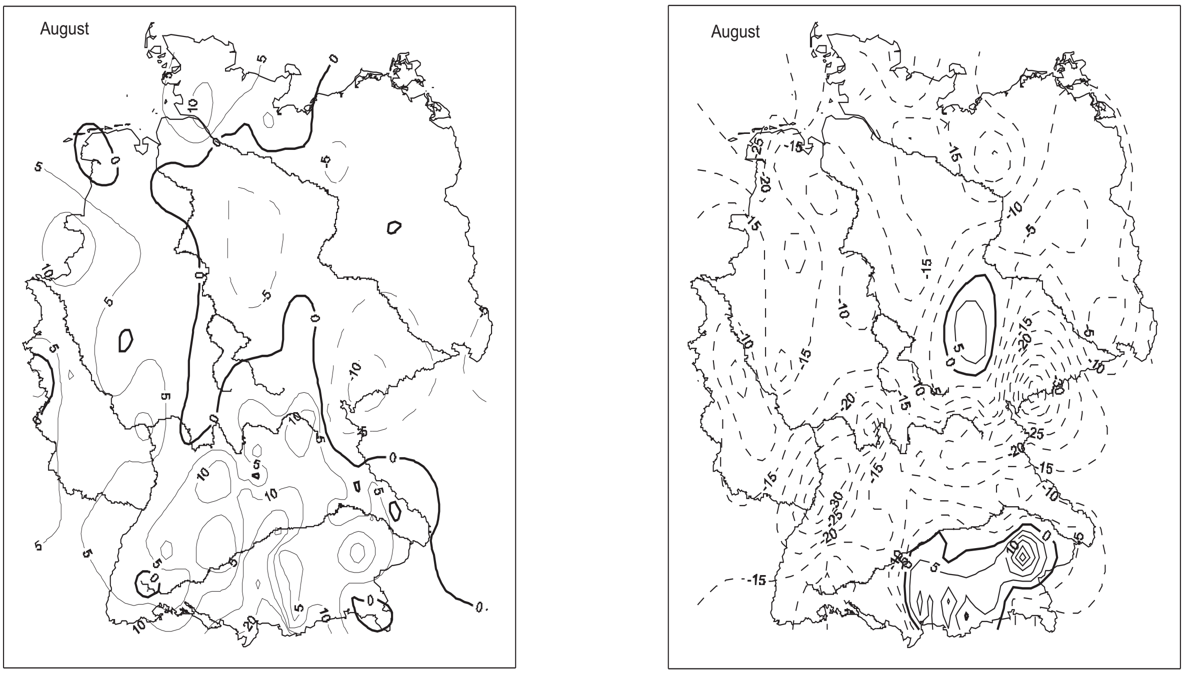

Figure 4: Same as in Figure 3 but for August.

ular, these simulations may quantify the differences in the results of the two approaches (least-squares and method introduced here) revealing the more robust estimator which should be preferred.

The residual analysis of the generalized time series decomposition technique (see again Trömel et al. (2005) for further details) confirms the complete description of the time series as a realization of a Gumbel-distributed random variable. Consequently, the least-squares estimator is applied to artificial generated Gumbel-distributed time series in order to evaluate the trend estimates on the basis of the least-squares method on the one hand and the trend estimates on the basis of the statistical modeling on the other hand. Generated random time series with a priori defined changes in the expected value are used to compare the given trends with the least-squares estimates. In this way a possible systematic bias associated with the application of the least-squares method to observed non-Gaussian precipitation time series can be identified.

Eight different experiments, i. e. positive and negative changes in the expected value caused by changes in the location and the scale parameter of the distribution, are undertaken. Set trend amplitudes are in the magnitude of monthly trends in precipitation time series in millimeter per year observed in the 20th century in Germany. The experiments only consider linear trends.

In the first step 100 Gumbel-distributed random time series with a sample size of N=1000 values are generated for each experiment, respectively. Table 1 shows for each experiment the actual change in the expected value, caused by linear changes in the location parameter Δa and the scale parameter Δb, the mean value $\overline{\Delta\mu}_{KQ}$ of all 100 least-squares estimates of the trend, determined by linear regression, as well as the standard deviation of these trends $\sigma_{\Delta\mu}$, the maximum $\Delta\mu^{+}_{KQ}$ and the minimum $\Delta\mu^{-}_{KQ}$ of all the trend estimates. Evidently, this Table shows only minor differences in the mean value $\overline{\Delta\mu}_{KQ}$ and a priori defined trend amplitudes. However, we observe a positive bias and the bias increases with the magnitude of the scale parameter. One possible explanation is that relatively high values are getting more weight than justified from the statistical point of view. The standard deviations $\sigma_{\Delta\mu}$ are considerably high. Depending on the occurence of the relatively high values in the Gumbel-distributed random time series, the least-squares estimator spreads in the first experiment with linear increment in the location parameter Δa=15 and unvaried scale parameter b=50 between 0.78 und 33.44. Furthermore, this range also increases with the value of the scale parameter. It is worth mentioning that in the case of an unvaried expected value (Δa=0, b=40) and even though in case of an increase of the expected value (Δa=8, b=40), the least-squares method sometimes provides negative trend estimates.

The sample size plays a major role for the mean squared error of the estimator, defined as the sum of the squared bias $(\overline{\Delta\mu}_{KQ} - \Delta\mu)^2$ and the variance $(\sigma_{\Delta\mu})^2$. So, all

Table 1: Trends of 100 generated Gumbel-distributed time series of sample size N=1000. The predefined linear trends are caused by linear changes in the location parameter Δa with simulaneous changes the scale parameter $\Delta b\Delta a$ or constant scale parameter b. Real changes in the expected value $\Delta\mu$ are compared to the mean value of least-squares estimates $\overline{\Delta\mu}_{KQ}$. Furthermore, the standard deviation $\sigma_{\Delta\mu}$ as well as the maximum and the minimum ($\Delta\mu^{+}_{KQ}$ bzw. $\Delta\mu^{+}_{KQ}$) of the trend estimates are given.

	Δa=15, b=50	Δa=15, b=20	Δa=8, b=40	Δa=0, b=40	Δa=8, b=40+Δ8	Δa=-15, b=50	Δa=-8, b=40-Δ8	Δa=0, b=40+Δ10
$\Delta\mu$	15	15	8	0	12.6	-15	-12.6	5.77
$\overline{\Delta\mu}_{KQ}$	15.49	15.20	8.39	0.39	13.05	-14.51	-12.27	6.22
$\sigma_{\Delta\mu}$	6.97	2.79	5.57	5.57	6.16	6.70	5.02	6.31
$\Delta\mu^{-}_{KQ}$	0.78	9.31	-3.38	-11.38	0.08	-29.22	-22.83	-7.06
$\Delta\mu^{+}_{KQ}$	33.44	22.38	22.75	14.75	28.23	3.44	1.27	21.81

experiments are performed again with the sample size N=100. Table 2 shows the results analogous to Table 1. We observe a negative bias now in all experiments. Furthermore, the absolute value of the bias is significantly greater compared to the experiments with sample size N=1000. Generally, the standard deviation or variance of the trend estimator is greater in case of smaller sample size. The trend amplitudes of the 100 generated time series of each experiment enclose a width range of variation. The experiment with a linear increase of 15 units in the location parameter and unvaried scale parameter provides trend amplitudes between -43.83 and 59.52.

The application of the least squares estimator for trend analyses of precipitation time

Table 2: Same as Table 1, but the sample size of the random time series is N=100.

	Δa=15, b=50	Δa=15, b=20	Δa=8, b=40	Δa=0, b=40	Δa=8, b=40+Δ8	Δa=-15, b=50	Δa=-8, b=40-Δ8	Δa=0, b=40+Δ10
$\Delta\mu$	15	15	8	0	12.6	-15	-12.6	5.77
$\overline{\Delta\mu}_{KQ}$	12.85	14.14	6.28	-1.72	10.78	-17.15	-14.21	3.90
$\sigma_{\Delta\mu}$	23.82	9.53	19.06	19.06	20.91	23.82	17.31	21.39
$\Delta\mu_{KQ}^{-}$	-43.84	-8.54	-39.07	-47.07	-38.96	-73.84	-55.18	-49.93
$\Delta\mu_{KQ}^{+}$	59.52	32.81	43.62	35.62	51.44	29.52	22.13	46.37

Table 3: With the distance funtion of the Gumbel distribution (Index G instead of KQ) estimated trends of 100 generated Gumbel-distributed time series of sample size N=1000. The experiments and notation is the same as in Table 1 and Table 2.

	Δa=15, b=50	Δa=15, b=20	Δa=8, b=40	Δa=0, b=40	Δa=8, b=40+Δ8	Δa=-15, b=50	Δa=-8, b=40-Δ8	Δa=0, b=40+Δ10
$\Delta\mu$	15	15	8	0	12.6	-15	-12.6	5.77
$\overline{\Delta\mu}_{G}$	15.56	15.20	8.5	0.47	13.27	-14.25	-12.23	6.21
$\sigma_{\Delta\mu_G}$	7.14	2.87	5.76	5.67	5.91	7.01	4.1	5.90
$\Delta\mu_{G}^{-}$	0.66	9.18	-3.57	-11.82	1.22	-29.20	-22.70	-6.24
$\Delta\mu_{G}^{+}$	32.22	21.87	25.08	13.73	27.50	1.60	-0.04	20.33

series seems to be problematic keeping in mind the great mean squared error. However, it is interesting to see the mean squared error of the trend estimator using the distance function of the Gumbel distribution ρ_G. Smaller mean squared errors are anticipated using the adequate distance function. The experiments presented in Table 1 and Table 2 are performed again and the distance function of the Gumbel distribution is used now. Statistical trend tests are not applied in order to retain comparability. A modified F-Test is necessary for non-Gaussian residuals (see Trömel, 2004). Table 3 and 4 show again for all experiments the mean value of the trend estimates $\overline{\Delta\mu}_G$, the standard deviation $\sigma_{\Delta\mu_G}$, the minimum $\Delta\mu_G^-$ and maximum $\Delta\mu_G^+$ arising in 100 generated time series, respectively. Again experiments are performed using the sample size N=1000 and N=100. Obviously, application of the adequate distance function is not associated with a smaller mean squared error of the trend estimator. It is interessting to see that not the distance function ρ but the sample size N is the most important factor to achieve reliable estimators for trends of precipitation time series. Generally, consistency is a property of the maximum likelihood estimator. A possible explanation for similar mean squared errors may be the significance

Table 4: Same as Table 3, but the sample size of the random time series is N=100.

	Δa=15, b=50	Δa=15, b=20	Δa=8, b=40	Δa=0, b=40	Δa=8, b=40+Δ8	Δa=-15, b=50	Δa=-8, b=40-Δ8	Δa=0, b=40+Δ10
$\Delta\mu$	15	15	8	0	12.6	-15	-12.6	5.77
$\overline{\Delta\mu}_G$	11.18	13.43	5.06	-2.88	9.39	-18.24	-14.95	2.57
$\sigma_{\Delta\mu_G}$	23.48	9.40	18.71	18.72	20.48	23.54	16.88	20.91
$\Delta\mu_G^-$	-44.70	-9.49	-40.28	-47.66	-40.38	-74.08	-55.58	-48.27
$\Delta\mu_G^+$	57.51	31.81	41.37	33.73	49.43	28.16	19.56	43.13

of the trends, which was not considered in the Monte-Carlo simulations. The additional differences between the F-test and the modified F-test would complicate the comparison of the two trend estimators.

However, not until the application of the adequate distance function a complete analytical description of observed precipitation is possible. In this way, monthly trends can be estimated on the basis of the whole sample size instead of a separate analysis of sub-time series containing only one 12th of the whole sample size. This is not possible with the use of the least-squares estimator. Considerung the smaller sample sizes of each month, trend estimators are too sensitive to single random values.

In conclusion, the method introduced here represents a tool to robust trend estimation of non-Gaussian precipitation time series. It allows to take into account changes in different parameters of the distribution. Furthermore, the greater sample size is essential for robust results. Differences in time series occuring by chance should not rise in different estimators because they only represent different realization of a random variable with the same statistical features. It appears that in case of least squares even the sign of the trend must not be the true.

4 Conclusions

This paper introduces an alternative approach to estimate trends in observed precipitation time series. In the special case of 132 time series of monthly precipitation totals 1901-2000, from German stations, the interpretation as a realization of a Gumbel distributed random variable with time-dependent location parameter and time-dependent scale parameter reveals a complete analytical description of the time series. The deterministic part contains the annual cycle and its changes concerning the amplitude and phase shifts, trends up to the order 5 and low frequency variations. These structured components are detected in the location and the scale parameter of the Gumbel distribution, which describes the statistical part of the series. On the basis of the achieved complete description of the time series, the difference between the expected value in a special month in the last year and the first year of the observation period can be defined as the trend in the considered month.

In winter, both trend maps, on the basis of the least-squares estimator and on the basis of the Gumbel model, show the same spatial distribution of detected trends but the amplitudes are samller in the latter case. In summer, the differences are more pronounced. While the least-squares estimator shows negative trends in the overwhelming majority of stations, the time series decomposition technique does not detect negative trends in the southern part of Germany. A shift of the distribution to higher values describes the observational time series in the southern part of Germany.

So, the method introduced here represents the possibility to estimate reliably different seasonal trends in observed monthly precipitation time series. Changes in different parameters of the distribution can be taken into account, too. This speaks in the favour of the more extensive statistical modeling because observed precipitation time series often show changes in the variance as well as the shape of the distribution. Trends for various months are estimated using the whole time series instead of the sub-sample of a special month. Monte-Carlo simulations underline the smaller mean squared error associated with an increase of the sample size. It is surprising, that the least-squares estimator does not seem to be inferior to the estimator of the Gumbel model if the bias and the variance are compared for the same sample size. Smaller errors were anticipated using the adequate distance function, because the distance function of the Gumbel distribution takes the skewed distribution of precipitation time series as well as the more prominent tails into account. Furthermore, significant changes in the scale parameter are considered in trend analyses on the basis of the Gumbel model, too. A possible explanation for similar mean squared errors using the same sample size may be the significance of the trends, which was not considered in the Monte-Carlo simulations. The additional differences between the F-test, used for the least-squares estimator, and the modified F-test, used within the generalized time series decomposition technique, and further several possible tests like the Mann-Kendall statistic would derange the comparison of the two trend estimators considered.

Definitely, the least-squares method provides less robust estimates and generates greater differences in the trends of various months. These great leaps in estimated trend amplitudes of subsequent months are untrustworthy in the meteorological sense. Within the method introduced significant detected changes in the annual cycle of the distribution parameters may generate different trends in various months. However, these seasonal variations are not caused by single relatively high values. The combination of the distance function of the Gumbel distribution and the greater sample size leads to robust monthly trend estimates of observed precipitation time series and constrains the influence of single relatively high values.

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